REFRIGERATOR OPTIMAL SCHEDULING TO MINIMISE THE COST OF OPERATION

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The cost optimal scheduling of a household refrigerator is presented in this work. The fundamental approach is the model predictive control methodology applied to the piecewise affine model of the refrigerator.
The optimisation could not be solved using off-the-shelf tools, e.g. Multi-Parametric Toolbox, so a binary tree-based optimal scheduling algorithm has been developed for this problem.

Keywords: optimal scheduling, model predictive control, operating cost minimization, modelling, day-ahead market

1. Introduction

Environmental awareness is trendy in modern societies. Policy-makers tend to prefer green energy, e.g. solar, wind, etc. in order to decrease carbon footprints. The demand-and-supply-based nature of the electricity market has led to hourly electricity prices in day-ahead markets [1]. Most people seek the cheapest solution by using their appliances during low price periods, however, there are certain devices, e.g. refrigerators work all day long. In this case, the only opportunity lies in the optimal scheduling of the cooling periods. The prescribed inside temperature of the refrigerator makes the problem difficult to solve.

2. Problem Statement

In a day-ahead market, the service provider provides the hourly electricity price for the next day. The problem to be solved is the optimal scheduling of a given refrigerator based on the forthcoming electricity prices. As a constraint, the refrigerator temperature must be kept within given boundary values.

2.1. Energy Price

The service provider provides the electricity price for the next 24 hours (DAM: day-ahead market). *Fig.1* shows the electricity price [2] over a week, each line corresponds to the price of a day.
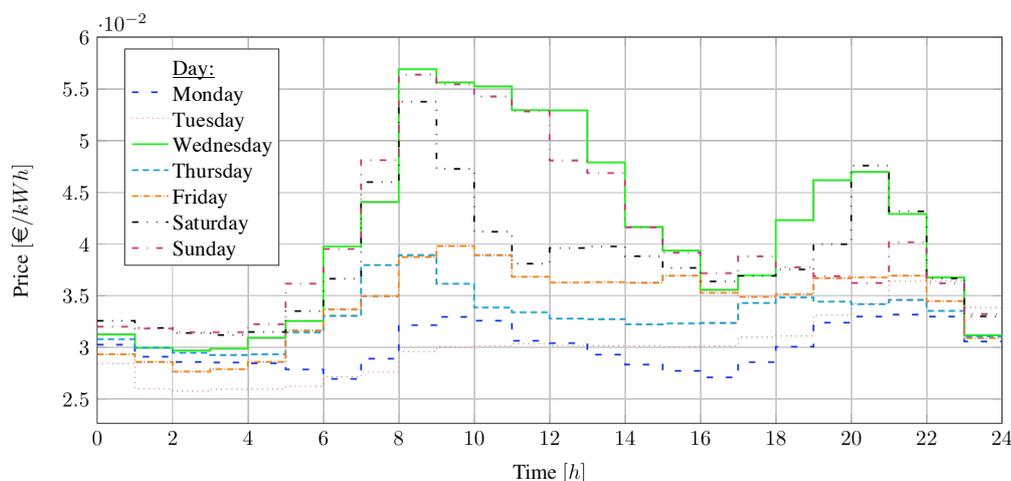

Figure 1. Electricity price of a day-ahead market over a week [2].

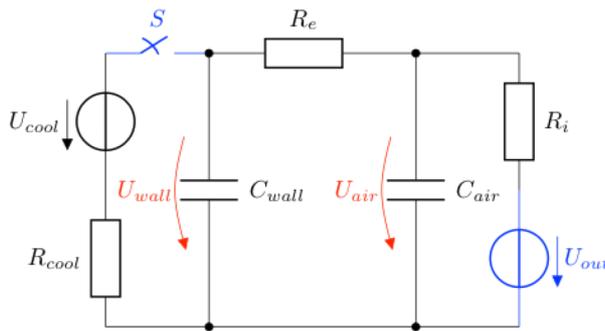*Correspondence: balint.roland@virt.uni-pannon.hu

Figure 2. Electric substitution model of a refrigerator. *S* symbolizes the switch of the devices controlled by a thermostat in an ordinary refrigerator.

2.2. Refrigerator Model

In order to describe the dynamics of the thermal behaviour of the refrigerator, the electric substitution model of *Fig.2* can be used, where the red voltages correspond to the inner air temperature (U_{air}) and the cooled back wall (U_{wall}). The blue elements are the inputs to the system: the outer air temperature (U_{out}) and the on/off switch (S). The capacitors demonstrate the heat capacity of the back wall (C_{wall}) and the inner air (C_{air}) while the resistors are the heat transfer coefficients between the components.

2.2.1. Cooling Dynamics

The first case is when the switch *S* is closed, i.e. the refrigerator is cooling. The dynamics of the system can be described by the state-space model of the form outlined in *Eqs.(1)* and *(2)*, where the state and input vectors are given in *Eqs.(3)* and *(4)*.

$$\dot{x} = \mathbf{A}\,x + \mathbf{B}\,u \qquad (1)$$

$$y = \mathbf{C}\,x + \mathbf{D}\,u \qquad (2)$$

$$x = \begin{bmatrix} T_{air} \\ T_{wall} \end{bmatrix} \qquad (3)$$

$$u = \begin{bmatrix} T_{out} \\ S \end{bmatrix} \qquad (4)$$

The parameters of the state-space model of the cooling dynamics are given as follows:

$$\mathbf{A}_{on} = \begin{bmatrix} -\dfrac{R_e + R_i}{R_e \cdot R_i \cdot C_{air}} & \dfrac{1}{R_e \cdot C_{air}} \\ \dfrac{1}{R_e \cdot C_{wall}} & -\left(\dfrac{1}{R_e \cdot C_{wall}} + \dfrac{1}{R_{cool} \cdot C_{wall}}\right) \end{bmatrix} \qquad (5)$$

$$\mathbf{B}_{on} = \begin{bmatrix} \dfrac{1}{R_i \cdot C_{air}} & 0 \\ 0 & \dfrac{U_{cool}}{R_{cool} \cdot C_{wall}} \end{bmatrix} \qquad (6)$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (7)$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (8)$$

2.2.2. Reheating Dynamics

The second case is when the switch *S* is open, i.e. the refrigerator is reheating to environmental temperature. The system is still linear and time invariant as outlined in *Eqs.(3)* and *(4)*. The parameters of the state-space model are given in *Eqs.(9)-(12)*.

$$\mathbf{A}_{off} = \begin{bmatrix} -\dfrac{R_e + R_i}{R_e \cdot R_i \cdot C_{air}} & \dfrac{1}{R_e \cdot C_{air}} \\ \dfrac{1}{R_e \cdot C_{wall}} & -\dfrac{1}{R_e \cdot C_{wall}} \end{bmatrix} \qquad (9)$$

$$\mathbf{B}_{off} = \begin{bmatrix} \dfrac{1}{R_i \cdot C_{air}} & 0 \\ 0 & 0 \end{bmatrix} \qquad (10)$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (11)$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (12)$$

2.3. Control Aim

The aim of this study is to schedule refrigerator's operation to minimise the operational costs with the following assumptions:
- the operational cost is the energy consumption of the refrigerator during the day;
- the price of electricity changes hourly;
- the energy price is known for 24 hours in advance;
- the temperatures must be between the following operating constraints:
  o the inner air temperature should be between 0.1 ℃ and 5.5 ℃;
  o the cooled back wall temperature should be between -19 ℃ and 7 ℃;
- the outer air temperature is constant;
- the input variable is binary (*On/Off*);
- the models should be chosen based on the value of the input.

The main objective is to minimize the operational costs, defined as

$$cost = \sum_{k=0}^{N-1} (p_k \cdot U_k) \qquad (13)$$

where p_k is the price of electricity and U_k is the input at time $T_s \cdot k$ (T_s: sample time).

3. Optimal scheduling

To find the optimal scheduling of a system the model predictive control approach is used. Afterwards a novel heuristic optimisation technique is presented.

3.1. Model Predictive Control of the System

For optimization, the Multi-Parametric Toolbox 3.0 (MPT) [3] is used first, because in this Matlab Toolbox we can find all the necessary options can be found:
- 'Piecewise Affine' (PWA) modelling for linear time invariant (LTI) systems;
- optimal controller design for PWA systems.

The solver uses a cost function as follows:

$$min \sum_{k=0}^{N-1}(||Q_x x_k||_p + ||Q_u u_k||_p), \qquad (14)$$

where variables x_k and u_k are the state and input vectors at the kth step of the prediction horizon N. The $||\cdot||_p$ expression is the standard p vector norm, and Q_x and Q_u are penalty matrices.

The MPT toolbox for the PWA system description uses discrete time models, thus initially the continuous time model must be discretised. However, there are two problems to solve:

- the penalty matrices (Q_x and Q_u) cannot be changed, so the actual electricity price cannot be used in this form;
- the time information of the system, e.g. system time, is unknown.

To handle these problems, two supplementary state variables have been introduced. The first counts the time and the second stores the actual energy price. If the refrigerator is switched off the energy price is zero, otherwise it provides the actual price value. One pair of (*On/Off*) systems for every price of each hour needs to be created, during the calculation the model parameters and the price are constants. Thus, the choice is based on the state of the binary input and the time value of the state vector. The modified LTI state-space models are the following:

$$x = \begin{bmatrix} T_{air} \\ T_{wall} \\ t \\ p \end{bmatrix} \qquad (15)$$

$$u = [\, S \,] \qquad (16)$$

$$\Phi_{on} = \begin{bmatrix} 0.9998 & 0.0001 & 0 & 0 \\ 0.0004 & 0.9977 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (17)$$

$$\Gamma_{on} = \begin{bmatrix} -0.0024 \\ -0.045 \\ 0 \\ p_h \end{bmatrix} \qquad (18)$$

$$\Phi_{off} = \begin{bmatrix} 0.9998 & 0.0001 & 0 & 0 \\ 0.0010 & 0.9988 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (19)$$

$$\Gamma_{off} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ p_h \end{bmatrix} \qquad (20)$$

$$f = \begin{bmatrix} 0.0022 \\ 0.028 \\ 1 \\ 0 \end{bmatrix} \qquad (21)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (22)$$

Table 1. The simulation time in minutes *versus* horizon size.

Size of horizon, -	8	10	12	13
Calculation time, min	2	8	59	111

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (23)$$

The hourly changing energy prices are denoted by p_h in matrices Γ, so there are 24 models with two system dynamics (*On* and *Off*) in total. The model actually used is chosen based on the input (*On* and *Off* states) and the third element of the state vector (t: define the model over time so the energy price value). Based on the two variables the necessary model is chosen:

- if the refrigerator operates, its value is the actual energy price;
- if the refrigerator is switched off, it is zero.

By the summation of the values of x_4 the operational cost is obtained (in €/kWhr) for the operating period. The piecewise affine model of the system is given by *Eq.(24)*.

$$x_{k+1} = \begin{cases} \Phi_{on} x_k + \Gamma_{on} u_k + f, & \text{if } [S] = 1 \\ \Phi_{off} x_k + \Gamma_{off} u_k + f, & \text{if } [S] = 0 \end{cases} \qquad (24)$$

The simulation is very sensitive to some design parameters. These parameters are the sampling time (T_S), the prediction horizon size, and the cycle number of the simulation horizon. The correlation between horizon size and simulation time is outlined in *Table 1* ($T_S = 5$ min). If the horizon is bigger than 12 – 15, the simulation will end in deadlock. The sample time value cannot be too large because the change in one step would be too high and the algorithm would not be able to function properly. The multiplication of the sample time and the horizon yields to predicted time horizon. If the sample time and the horizon are small, the algorithm runs quickly, but with a small pre-determined time interval and cannot calculate the forthcoming electricity prices. As N_{sim} increases, the running accelerates. In the PWA system, only the systems of the simulated time interval are present. If N_{sim} is small, the number of models is small so the algorithm must choose from fewer systems but in that case, the cycle has to run repeatedly as the new PWA system is defined.

3.2. Heuristic Search Algorithm using Binary Tree Growing

The general optimizer in the MPT is very slow due to several problem-specific assumptions, e.g. binary input, and the horizon size is bounded so another optimisation algorithm is needed that runs faster and uses problem-specific heuristics.
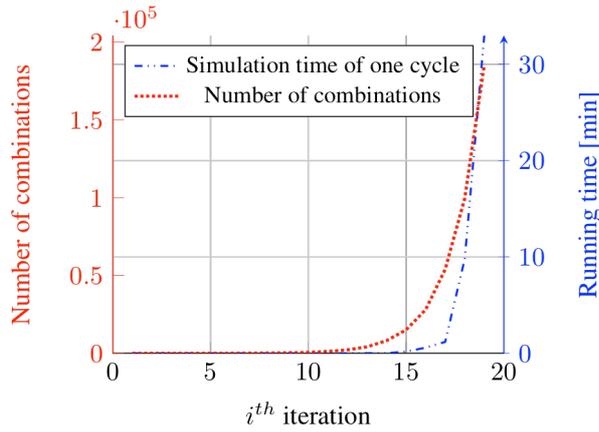
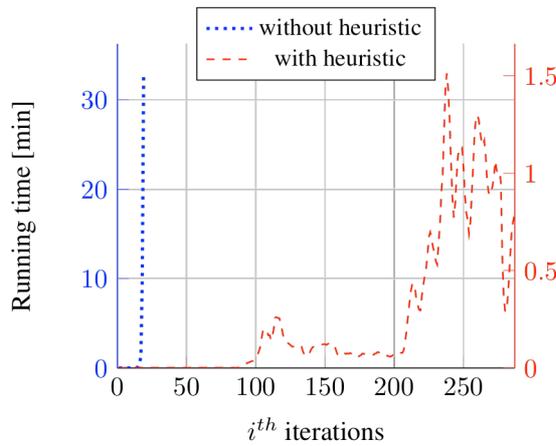*Figure 3*. Running time and number of combinations vs number of cycles.

Figure 4. Without the heuristic algorithm, the simulation time of the 20th cycle is more than 30 minutes, but with the heuristic algorithm, it has a maximum of 1.5 minutes.

3.2.1. Base of Algorithm

In order to find the optimal solution, all input combinations need to be counted over time. Trivially, this number can be reduced by deleting those combinations where the system exceeds the operating constraints. In this case, in all steps, the states are duplicated and 1 or 0 (*On* or *Off*) is added to input vectors so the number of combinations grow exponentially and the simulation time of one cycle grows, too (*Fig.3*). Without additional heuristics, the running time of the 18th loop was as high as the MPT simulation with a horizon size of 13.

Model-related heuristics can be applied to decrease the number of combinations. This rule is simple: between any two input combinations, if the corresponding air and wall temperatures are higher and also the operational cost is greater than or equal to one of the examined combinations then this input combination is not optimal, *i.e.* it can be ignored. The pseudo code of the algorithm is summarized by *Algorithm 1*. With these heuristics, the number of combinations and the running time of the simulation can be reduced. *Fig.4* shows the running time of the nth cycle with and without this filtering. This figure shows that without the filter, the simulation time of the 20th

cycle is more than 30 minutes but with the filter it is a maximum of 1.5 minutes.

Algorithm 1 Heuristic algorithm

  procedure OPTIMUM SEARCHING
  define:
    $X_0 \leftarrow$ initial state
    bounds ← bounds of temperatures
    $p_h \leftarrow$ electricity prices of a day
    $h = 1$ (actual hour of day)
    $T_s \leftarrow$ sample time [hour]
    SimTime ← simulation time
    cost = 0
    sys1 ← reheating DT-LTI system
    sys2 ← cooling DT-LTI system
    $U_{0,0} =[]$
  create:
    PWA ← PWA system from *sys1* and *sys2*
  loop:
    for $i = 0 : 1 : SimTime/T_s$ **do**
      $U_{i,all} = [U_{i-1,all}; U_{i-1,all}]$ (duplicate *U*)
      $n = size(U)$ (number of combinations)
      $k \in \{1, 2, ..., n\}$
      if $1 < k <= n/2$ **then**
        $U_{i,k} = [U_{i,k}, 1];$
      else
        $U_{i,k} =[U_{i,k}, 0];$
      $X_{i+1,k} = PWA(X_{i,k}, U_{i,k})$
      cost ← update *cost*
      $k \in \{1, 2, ..., n\}$
      if $X_{i+1,k} \in / bounds$ **then**
        delete $X_{all,k}, X_{all,k}$ and $cost_k$
      $n = size(U)$
      $k,l \in \{1, 2, ..., n\}$
      if $X_{i+1,k} > X_{i+1,l}$ **and** $cost_k > cost_l$ **then**
        delete $X_{all,k}, X_{all,k}, cost_k$
      else
        if $X_{i+1,l} > X_{i+1,k}$ **and** $cost_l > cost_k$ **then**
          delete $X_{all,l}, X_{all,l}, cost_l$
  $n = size(U)$
  $k \in \{1, 2, ..., n\}$
  $opt = min(cost_k)$
  optimal solution = $U_{all,opt}$

3.2.2. Algorithm with Horizon

The algorithm of the MPT uses a horizon to reduce the calculation time for fast online implementation. In the investigated problem, the maximum usable horizon available in MPT is too small. The previous algorithm calculates the optimal solution during a whole day, so the horizon is 24 hour, e.g. if the sample time is 5 minutes then the size of the horizon is 288. This large horizon size causes slow simulations. To decrease this simulation time, a horizon as in the MPT algorithm is defined. This step causes a very large reduction in running time. This time is proportional to the horizon size, but if the horizon is too small the solution would not be optimal. The running times and calculated costs are collected in *Table 2*.

The running times in *Table 2* show that by increasing the horizon size the simulation time increases, too. The operating cost increases, but it is uncertain whether a bigger horizon is better. Based on this, the suggested heuristic search-based optimization algorithm can be used as a basis for a special model predictive scheduling/control algorithm.

4. Results and Discussion

The model predictive control algorithm in the MPT did not work correctly due to the serious limitations connected to the control task. By increasing the horizon size, the simulation becomes slow with maximum horizon size of 13.

The proposed optimization algorithm has been developed to find the optimal scheduling. *Table 2* shows the difference between simulation times and operating costs if the horizon size is changed. *Fig.5* shows the calculated optimal scheduling for Wednesday. *Fig.6* shows the energy prices and the number of input

Table 2. The size of horizon, the simulation time in minutes and the cost of best solution.

Size of horizon	Running time	Cost	Deviation from optimal
none (∞)	75.0	0.20235	0%
24	5.7	0.20427	+0.95%
36	12.6	0.20433	+0.98%
48	16.9	0.20394	+0.79%
60	25.9	0.20413	+0.88%
72	31.0	0.20456	+1.09%

combinations of the ith cycle without and without a horizon size of 4 hours. There is a correlation between the monotonous energy price and the number of input combinations. If the price is increased the number opf combinations increases, too.

Simulations of energy prices for a whole week with and without horizon size of 4 hours ($T_S = 5$ minutes) have also been performed. The results are shown in *Table 3*. The simulation times were reduced by 70-90% while the costs grew only by 1-2%.
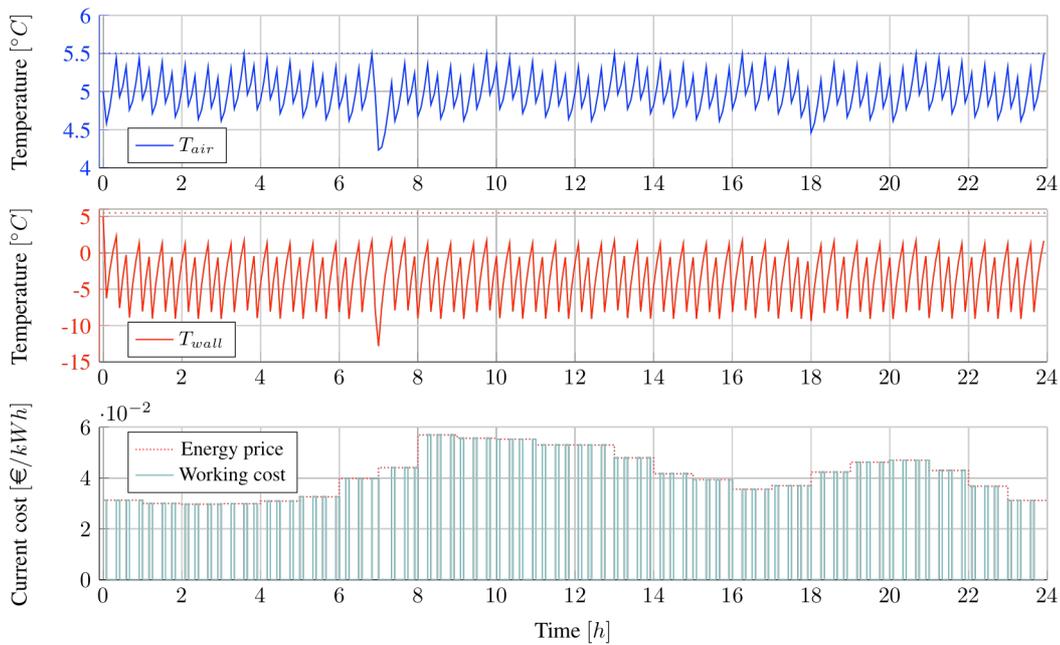
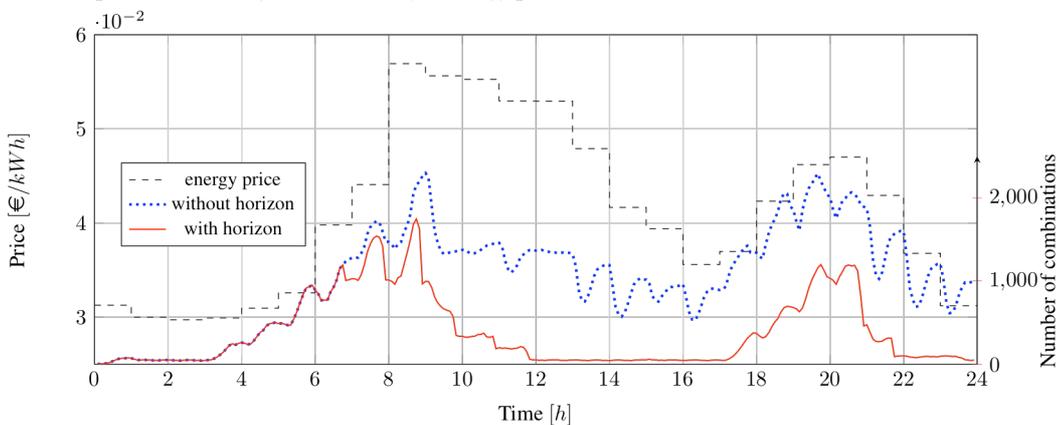*Figure 5.* The optimal scheduling for Wednesday's energy prices.

Figure 6. Wednesday energy prices and the number of input combinations of the ith cycle with and without a horizon size of 4 hours.

Table 3. Differences in optimal operating cost and simulation time with and without a horizon.

Day of the week	without horizon		4 hour horizon		difference in percent	
	cost, 1/kW	time, min	cost, 1/kW	time, min	cost, 1/kW	time, min
Monday	0.20235	75	0.20394	16.9	+0.79	-77.47
Tuesday	0.20200	1013.2	0.20650	78.9	+2.23	-92.21
Wednesday	0.27748	234	0.28111	48.8	+1.31	-79.18
Thursday	0.22255	116.1	0.22739	38.3	+2.17	-67.04
Friday	0.23127	267.7	0.23618	32.3	+2.12	-87.93
Saturday	0.25819	222.3	0.26281	45.1	+1.79	-79.70
Sunday	0.27286	100.4	0.27488	18.7	+0.74	-81.34

5. Conclusion

A heuristic search algorithm has been implemented for the optimal scheduling of a refrigerator model. This work was initiated by the fact that the optimization algorithm of the MPT did not work correctly with large horizon sizes for the applied piecewise affine system. The proposed algorithm calculates the optimal input sequence for a whole day with any horizon size but it also works with an infinitely large horizon. The calculation time is fast compared to that of the Matlab MPT solver.

Our own heuristic algorithm is faster without a horizon than the MPT solver with horizon size of 13 hours. I can be run with a larger horizon size, and can yield the absolute optimum without a prediction horizon. The operating cost of the solution is 12 percent less with the aid of a heuristic search. This cost is 25 percent cheaper than the cost of traditional refrigerator control.

The suggested heuristic search-based optimization algorithm can be used as a basis of a special model predictive scheduling/control algorithm. The present work is based on an ideal refrigerator model. As a future step, the validation of the model and the scheduling algorithm in an MPC framework will be investigated. Testing the control scheme in a real situation with random disturbances, e.g. opening the refrigerator's door, etc. will also be performed.

SYMBOLS

A	state matrix
B	input matrix
C	output matrix
C_{air}	heat capacity of the inside of the refrigerator
C_{wall}	heat capacity of the back-wall
DAM	Day-Ahead Market
DT-LTI	Discrete Time Linear Time Invariant
f	constant input or disturbance
Φ_{off}	off state discrete time state matrix
Φ_{on}	on state discrete time state matrix
Γ_{off}	off state discrete time input matrix
Γ_{on}	on state discrete time input matrix
LTI	Linear Time Invariant
MPC	Model Predictive Control
MPT	Multi-Parametric Toolbox

PWA	Piecewise Affine
Q_u	penalty matrix of u vector in MPT
Q_x	penalty matrix of x vector in MPT
R_{cool}	heat resistance of the back-wall
R_e	heat resistance between back-wall and inner air
R_i	insulation of refrigerator
S	binary state switch
T_{air}	inner air temperature of refrigerator
T_{out}	outer room temperature
T_{wall}	cooled back-wall temperature of refrigerator
u	input vector
u_k	input vector value in k^{th} sample moment in DT
U_{air}	voltage corresponding to inner air temperature
U_{cool}	voltage corresponding to minimum reachable back-wall temperature
U_{out}	voltage corresponding to outer air temperature
U_{wall}	voltage corresponding to cooled back-wall temperature
x	state vector
x_k	state vector value in k^{th} sample moment in DT

REFERENCES

[1] Huisman, R.; Huurman, C.; Mahieu, R.: Hourly electricity prices in day-ahead markets, *Energy Econ.* 2007 **29**(2), 240–248 DOI 10.1016/j.eneco.2006.08.005

[2] Nord Pool: Elspot prices, www.nordpoolspot.com/Market-data1/Elspot/Area-Prices/ALL1/Hourly, 2016

[3] Herceg, M.; Kvasnica, M.; Jones, C.; Morari, M.: Multi-parametric toolbox 3.0, *Proc. Eur. Control Conf.*, EPFL-CONF-186265 (Zurich, Switzerland) 2013