HUNGARIAN JOURNAL
OF INDUSTRIAL CHEMISTRY
VESZPRÉM
Vol. 30. pp. 13 - 18 (2002)

ON THE NON-LINEAR MASS TRANSFER THEORY

C. BOYADJIEV

(Institute of Chemical Engineering, Bulgarian Academy of Sciences, "Acad. G.Bontchev" str.,Bl.103, 1113 Sofia, BULGARIA)

Received: February 19, 2001

A theoretical analysis of non-linear mass transfer kinetics has been done. The comparison between Stephan flow, a flow induced by large concentration gradients and Marangoni effect is shown. The conditions of existing of these non-linear effects in mass transfer kinetics are determined.

Keywords: non-linear effects, Stephan flow, Marangoni effect

Introduction

A theoretical analysis of non-linear mass transfer has been developed in [1]. The main idea follows from the non-linearity of the convection-diffusion equation:

$$\rho(c)\boldsymbol{W}(c)\mathbf{grad}c = div\left[\rho(c)D(c)\mathbf{grad}c\right]+kc^n . \quad (1)$$

The velocity $\boldsymbol{W}$ is governed by the hydrodynamic equations. However, the principal non-linear phenomenon is due to the concentration effects on the velocity $\boldsymbol{W}(c)$, density $\rho(c)$, viscosity $\mu(c)$, diffusivity $D(c)$ and on the chemical reaction rate kc^n (for $n \neq 1$).

It was shown [1] that there are a number of cases with non-linear mass transfer behavior. The well-known linear mass transfer theory could be successfully applied in these cases. However, in case of two-phase interphase mass transfer with a flat interface the above equation permits a non-linear mass transfer model to be derived by means of the boundary layer approximation:

$$\rho_i\left(u_j\frac{\partial u_j}{\partial x}+v_j\frac{\partial u_j}{\partial y}\right)=\mu_j\frac{\partial^2 u_j}{\partial y^2}+A_i , \quad \frac{\partial u_j}{\partial x}+\frac{\partial v_j}{\partial y}=0 , \quad (2)$$

$$u_j\frac{\partial c_j}{\partial x}+v_j\frac{\partial c_j}{\partial y}=D\frac{\partial^2 c_j}{\partial y^2}+B_j , \quad j=1,2;$$

$$x=0, \quad u_j=u_{j0} , \quad c_j=c_{j0};$$

$$y=0, \quad u_1=u_2 , \quad \mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y} , \quad v_j=0;$$

$$c_1=\chi c_2 , \quad D_1\frac{\partial c_1}{\partial y}=D_2\frac{\partial c_2}{\partial y};$$

$$y=\left(-1\right)^{j+1}\infty, \quad u_j=u_{j0} , \quad c_j=c_{j0}; \quad j=1,2 ,$$

where the index *1* is used to denote gas or liquid phase, while the index *2* designates liquid or solid phase. The terms A_j and B_j (j=1,2) are the contributions of some additional physical effects.

The solution of the above set of equations permits the interphase mass transfer rate to be determined as follows:

$$J=MK_1\left(c_{10}-\chi c_{20}\right)=\frac{MD_1}{L}\int_0^L\left(\frac{\partial c_1}{\partial y}\right)_{y=0}dx= \\ =MK_2\left(\frac{c_{10}}{\chi}-c_{20}\right)=\frac{MD_2}{L}\int_0^L\left(\frac{\partial c_2}{\partial y}\right)_{y=0}dx . \quad (3)$$

There are a number of processes where u_j, v_j, μ_j, ρ_j, D_j, A_j, and B_j are independent on the concentration c_j (j=*1,2*). These situations are the basis of the linear mass transfer theory.

The mathematical model allows the following principle characteristics of the linear mass transfer to be drawn:

the interphase mass transfer rate J does not depend on the mass transfer direction;

Contact information: E-mail: chboyadj@bas.bg

the interphase mass transfer coefficient K_j does not depend on the concentrations c_{j0} $(j=1,2)$.

The striving to decrease the size of the industrial devices necessitates process intensification. The systems with intensive mass transfer are characterized by a behavior that deviates considerably from the characteristics mentioned above. The main feature is the higher mass transfer rate, which differs significantly from the value predicted by the linear mass transfer theory. The non-linear effects leading to above have been described [1].

In systems with high concentrations and such exhibiting large concentration gradients, the deviations from the linear Fick's diffusion law are significant, too. Under such conditions, the higher concentrations could affect both the viscosity and the density of the fluid:

$$D_j = D_j(c_j), \quad \nu_j = \mu_j(c_j), \quad \rho_j = \rho_j(c_j), \quad j=1,2\,. \tag{4}$$

The concentration effect introduces a non-linearity in the convection-diffusion equation discussed in details in [2,3].

The other non-linear effect due to the non-uniform concentration distributions

$$A_j = g(\rho_j - \rho_{0j}), \quad \rho_j = \rho_j(c_j) \tag{5}$$

leads to a natural convection [4,5].

The next cause that may intensify the mass transfer process is the existence of a chemical reaction with a rate B_j in the bulk of the phase:

$$B_j = B_j(c_j), \quad j=1,2\,. \tag{6}$$

The studies developed in [1, 6], show that in the gas-liquid systems with a chemical reaction: $B_1=0$, while $B_2=kc^n$. Moreover, the chemical reaction rate could affect significantly the interphase mass transfer mechanism.

The thermal effect of the chemical reactions could lead to the temperature non-uniformity on the interface and to consequent surface tension gradients. This calls for new boundary conditions taking into account the equality of the tangential components of the stress tensor on the interface:

$$y=0, \quad \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} - \frac{\partial \sigma}{\partial x}\,. \tag{7}$$

The investigation of this effect (Marangoni effect) [7,8] has shown that it is negligible when there are no surfactants in the system.

One of the most interesting non-linear effects arises from the conditions imposed by the high concentration gradients. The latter induce secondary flows at the interface. This effect has been discussed in details in [1] for a large number of systems taken as examples and it has been termed "non-linear mass transfer effect ".

Under the conditions imposed by high concentration gradients induced secondary flows occur. They provoke convective components of the mass transfer flux additional to the main mass flux. In this case the mass transfer rate is:

$$J = \frac{MD\rho^*}{L\rho_0} \int_0^L \left(\frac{\partial c}{\partial y}\right)_{y=0} dx\,, \tag{8}$$

where the secondary flow affects both the diffusion mass transfer $D\left(\frac{\partial c}{\partial y}\right)_{y=0}$ and the convective mass transfer $\frac{M\rho^*}{\rho_0}$. In gas-liquid and liquid-liquid systems [9, 10] the non-linearity has been proven to be the effect of the induced secondary flow on the diffusion transfer. In liquid-solid systems the induced flow affects mainly the convective transfer. These effects are clearly demonstrated the electrochemical systems [1, 6] due to the high molecular mass (M) of the metals.

All the non-linear effects influence the velocity fields, which lead to changes in the hydrodynamic stability of the system. The loss of stability could cause an increase of the amplitudes of the random disturbances until a new stable state or a stable periodic process is reached [1]. The latter is a self-organizing dissipative structure with a mass transfer rate growing sharply which is not the case in the conventional systems. The problem has been discussed in details in [4, 5] in the case of non-stationary absorption of pure gases in immobile liquid layer with flat interface.

Non-linear effects in mass transfer kinetics induced by the secondary flows, lead to quality changes in the mass transfer rate, because they are related to physical mechanisms.

In this sense the most interesting are the Stephan flow [11], (occurred by volume change at the interface), the flow induced by large concentration gradients [1,12] and the flow resulted from surface tension gradients [7,8]. These secondary flows require introduction of new boundary conditions at the interface between gas – solid, gas – liquid, liquid – liquid and liquid – solid phases. A comparative analysis of the occurrence of these secondary flows will be presented.

Stephan Flow

The Stephan flow occurs in case of heterogenic reactions at the interface between two phases. As a result the substances are disappeared (reagents) or generated (reaction products) at this interface. This "disappearance or generation" of substances at the boundary between two phases might be a consequence of chemical reactions, adsorption or desorption processes (the substances connect physically or chemically with substances from phase boundary), interphase mass transfer (the substances "disappear" at phase boundary as a result of the transfer into the other phase), liquid – vapour phase transition (boiling, condensation). Some of the above heterogenic reactions lead to changes in the volume of the phase at the interface, which leads to occurrence of hydrodynamic flow called Stephan flow.

Let's consider the heterogenic reaction [11], presented by the stoichiometric equation:

$$\sum_{i=1}^{N} \upsilon_i A_i = 0\,, \qquad (9)$$

where A_i and $\upsilon_i (i = 1,...,N)$ correspond to the symbols of participating substances in the reaction and their stoichiometric coefficients. For the initial substances (reagents) $\upsilon_i > 0$, while for the reaction products $\upsilon_i < 0$. The rate of the heterogenic reaction $j_i (mol/m^2 s)$ is defined for the separate substances $(i = 1,...,N)$, where N is their total number. For the reagents $j_i > 0$, and for the reaction products $j_i < 0$.

The reagents (reaction products) are supplied to (took off from) the reaction surface by diffusion and convection:

$$\mathbf{j} = -D\mathbf{grad}\, c + \mathbf{v}c\,, \qquad (10)$$

where j is the vector of the mass transfer rate , $D(m^2/s)$ is the diffusivity, **grad** - is the vector of the gradient, $c(mol/m^3)$ molar concentration, $\mathbf{v}$ - velocity vector. For the separate substances the molar flux has the following form:

$$\mathbf{j}_i = -D_i \mathbf{grad}\, c_i + \mathbf{v}c_i, \quad i = 1,...N\,. \qquad (11)$$

The projection of the vectors in the vector equation over the normal vector of the interface $\boldsymbol{n}$ (in points of the surface) might be noted as:

$$j_i = (\mathbf{j}_i.\mathbf{n}), \quad \frac{\partial c_i}{\partial n} = (\mathbf{grad}\, c_i.\mathbf{n}), \quad v = (\mathbf{v}.\mathbf{n}), \qquad (12)$$

where j_i $(mol/m^2 s)$ are molar fluxes, which have to be equal to the rates of the reactions of the separate substances, and $\frac{\partial c_i}{\partial n}$ - normal derivate at the interface, v - the rate, induced as a result of heterogenic reaction at certain conditions and is called the velocity of the Stephan flow. It is positive when $\mathbf{v}$ is oriented to the phase boundary and negative in the opposite case.

The introduction of (12) into (11) leads to:

$$j_i = -D_i \frac{\partial c_i}{\partial n} + v c_i, \quad i = 1,...,N\,, \qquad (13)$$

where $j_i (i = 1,...,N)$ should be satisfy the condition for the stoichiometry of the flows:

$$j_1/\upsilon_1 = j_2/\upsilon_2 = ... = j_N/\upsilon_N\,. \qquad (14)$$

From (14) can be seen, that the stoichiometry coefficients $\upsilon_i (mol/m^2 s)$ present mols of the substances $(i = 1,...,N)$, which participate in the heterogenic reaction (per unit area, per unit time). They can be presented as:

$$\upsilon_i = v_i / w_i, i = 1,...,N\,, \qquad (15)$$

where $v_i (m^3/m^2 s)$ is the volume reaction rate of the substances in the gas (vapour) phase, and $w_i (m^3/mol)$ - their molar volume.

The systems gas (vapour) – liquid (solid) will be considered below, where the Stephan flow occurs in gaseous (vapour) phase, because it is practically not physically applicable in liquid and solid phases.

The summation of the stoichiometric coefficients leads to:

$$\upsilon = \sum_{i=1}^{N} \upsilon_i\,, \qquad (16)$$

where $\upsilon > 0 (\upsilon < 0)$ means the increase (decrease) of the mols number (the volume) of the reaction mixture as a result of the heterogenic reaction.

From (14) directly follows:

$$j_i = \frac{\upsilon_i}{\upsilon_1} j_1, \quad i = 1,...,N\,. \qquad (17)$$

The summing up of (17) leads to:

$$\sum_{i=1}^{N} j_i = \gamma j_1, \quad \gamma = \frac{\upsilon}{\upsilon_1}, \qquad (18)$$

where the substance A_1 we consider as limiting, i.e. the rate of its reaction limits rate of the heterogenic reaction.

In case of gases and vapours we can express the concentration through the partial pressure:

$$c_i = \frac{P_i}{RT}, \quad i = 1,...,N\,, \qquad (19)$$

where R is the universal gas constant, T - temperature. In this way from (13) directly follows:

$$j_i = \frac{D_i}{RT} \frac{\partial P_i}{\partial n} + \frac{vP_i}{RT}, \quad i = 1,...,N\,. \qquad (20)$$

The summation of (20) leads to:

$$\sum_{i=1}^{N} j_i = -\frac{1}{RT} \sum_{i=1}^{N} D_i \frac{\partial P_i}{\partial n} + \frac{vP}{RT}, \qquad (21)$$

where $P = \sum_{i=1}^{N} P_i$ is the total pressure of the mixture.

The velocity of the Stephan flow is obtained directly from (18) and (21):

$$v = \frac{RT}{P} \gamma j_1 + \frac{1}{P} \sum_{i=1}^{N} D_i \frac{\partial P_i}{\partial n}\,. \qquad (22)$$

In case of two component mixtures $D_1 = D_2 = D$, i.e.:

$$v=\frac{RT}{P}\gamma j_1+\frac{D}{P}\frac{\partial P}{\partial n}. \qquad (23)$$

It can be seen from (23) that the velocity of the Stefan flow is determined by the relative change in the volume of the reaction mixture γ as a result from changes in the volume velocity v_i or in case of phase transition (the change of the molar volume w_i). This velocity decreases as a result of hydraulic resistance $\left(\frac{\partial P}{\partial n}<0\right)$. At absence of the hydraulic resistance $P=const$ and the velocity of Stephan flow takes the form:

$$v=\frac{RT}{P}\gamma j_1. \qquad (24)$$

In case of reduction of the reaction mixture volume as a result of heterogenic reaction the Stephan flow is oriented to the reaction interface ($\gamma>0, v>0$). In opposite case ($\gamma<0, v<0$) it is oriented from the reaction interface.

In case of heterogenic chemical reaction without phase transition:

$$w_1=w_2=...=w_N, \gamma=\frac{\sum_{i=1}^{N} v_i}{v_1}, \qquad (25)$$

i.e. $v>0$, when the total volume rate of the chemical reaction of the substances in the mixture is positive and the volume increases. In opposite case $v<0$.

In cases when the heterogenic reaction presents the liquid–vapour phase transition at the interface (boiling, condensation) the molar rates and (as a results) their volume rates are equal:

$$v_1=v_2, \quad \gamma=\frac{\frac{1}{w_1}-\frac{1}{w_2}}{\frac{1}{w_1}}=\frac{w_2-w_1}{w_2}, \qquad (26)$$

i.e. in case of condensation (boiling) $w_1>w_2, \gamma<0$, $(w_2>w_1, \gamma>0)$ and the Stephan flow is oriented to (from) the interface.

In cases, when the heterogenic reaction presents adsorption (desorption) $v_1=v_2, \gamma=0$, i.e. the conditions for the Stephan flow do not exist. The analogous is the case of absorption (desorption), where the product transfers into the second phase, i.e. $v_1=v_2, v=0$.

The obtained result (24) shows that Stephan flow at interface arises when the heterogenic reaction leads to changes in the summary (total) volume of reaction mixture. Obviously this can happen only at phase boundary in gas or vapour phase and it is practically impossible at the boundary in liquid or solid phase.

Non - linear mass transfer

One of the significant effects of non- linear mass transfer arises in systems with intensive interphase mass transfer, when large concentration gradients induce secondary flows which velocity is normally oriented to the interface.

For a simplicity we will consider two-component fluid [1, 6], where the component A is a substance dissolved into component B (solvent). The density of the solution $\rho\ (kg/m^3)$ can be presented through the mass concentrations of the component $A\ (Mc)$ and solvent $B\ (M_0c_0)$:

$$\rho=M_0c_0+Mc=\rho_0+Mc, \qquad (27)$$

where M and M_0 are the molar mass (kg/mol) of A and B, c and c_0 - their molar concentrations (mol/m^3).

Every elementary volume of the solution has a velocity $\mathbf{V}$, which can be expressed through velocities of the substances $A(\mathbf{v})$ and $B(\mathbf{v}_0)$, i.e velocity of the mass flow, transferred by every elementary volume presents a sum of the mass flows of A and B:

$$\rho\mathbf{V}=\rho_0\mathbf{v}_0+Mc\mathbf{v}. \qquad (28)$$

The equation (28) can be projected over the normal vector $\mathbf{n}$ of the phase surface:

$$\rho^*(\mathbf{V}.\mathbf{n})=\rho_0^*(\mathbf{v}_0.\mathbf{n})+Mc^*(\mathbf{v}.\mathbf{n}), \qquad (29)$$

where the upper index $(^*)$ notes the values at the interface.

From (29) the velocity of the secondary flow $v(m/s)$ induced by the diffusion (large concentration gradient) can be determined:

$$v=(\mathbf{V}.\mathbf{n}). \qquad (30)$$

At the boundary between two non-mixing phases the mass flux is zero, i.e.

$$(\mathbf{v}_0.\mathbf{n})=0. \qquad (31)$$

The molar flux of the dissolved substance (at interface) $N\ (mol/m^2s)$ can be expressed through the rate (mol/m^2s) of the diffusion and convective transfer:

$$N=c^*(\mathbf{v}.\mathbf{n})^*=-D\left(\frac{\partial c}{\partial n}\right)^*+vc^*. \qquad (32)$$

The introduction of (30), (31) and (32) into (29) leads to:

$$v=-\frac{MD}{\rho_0^*}\left(\frac{\partial c}{\partial n}\right)^*, \quad \rho_0^*=M_0c_0^*, \qquad (33)$$

where c_0^* is the molar concentration of B at the interface. For a flat phase boundary $y=0$ is obtained directly [1, 6]:

$$v=-\frac{MD}{\rho_0^*}\left(\frac{\partial c}{\partial y}\right)_{y=0}. \qquad (34)$$

In some approximations [12] N is expressed only by the diffusion flux:

$$N=-D\left(\frac{\partial c}{\partial y}\right)_{y=0} \qquad (35)$$

and for the velocity of the secondary flow is obtained

$$v=-\frac{MD}{\rho_0^*}\left(\frac{\partial c}{\partial y}\right)_{y=0}, \quad \rho^*=\rho_0^*+Mc^*. \qquad (36)$$

Obviously the obtained results (34) and (36) coincide at $c^*=0$ (for example at desorption of gases).

From (34) it is seen that in the systems with an intensive interphase mass transfer the normal component of the velocity differs from zero (as in the systems with linear mass transfer), and depends on the concentration of the transferred substance, i.e. the convection–diffusion equation is non linear. This requires the boundary condition at $y=0$ ($v=0$) to be replaced with (34).

The obtained result (34) shows that the local mass flux at phase boundary has diffusion and convective components:

$$i=-MD\left(\frac{\partial c}{\partial y}\right)_{y=0}+Mvc^*=-MD\frac{\rho^*}{\rho_0^*}\left(\frac{\partial c}{\partial y}\right)_{y=0}. \qquad (37)$$

From (37) the mass transfer rate can be directly determined by averaging of the mass flux $i\left(kg/m^2s\right)$ over the interface.

The comparison between Stephan flow's velocity (24) and the velocity of the secondary flow induced by large concentration gradients shows that Stephan flow arises in gaseous or vapour phase as a result of the changes in the phase volume at the interphase. Such changes occur in some heterogenic chemical reactions accompanied by changes of the reaction mixture volume or phase transition (boiling, condensation).

The secondary flow at large concentration gradients is a result of the intensive interphase mass transfer and can be observed in gaseous and liquid phase (the Stephan flow in liquid phase is physically impossible).

Marangoni effect

The Marangoni effect is a result of secondary flow which velocity is tangentially oriented to the interface and is caused by the surface tension gradient, induced by concentration or temperature gradient at the interface. This effect will be considered in gas – liquid systems.

The influence of the secondary flows over the mass transfer rate is a result of the velocity component oriented normally to the interface. This creates an intensive convective transfer, which is summarized with the diffusion transfer. At the Marangoni effect the induced flow is tangential and the normal component appears from the flow continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \qquad (38)$$

The flows in the boundary layer are characterized by two characteristic scales of velocity (u_0, v_0) and two linear scales (δ, L), which are related to the dimensionless variables of the flow:

$$u=u_0U, \quad v=v_0V, \quad x=LX, \quad y=\delta Y, \quad \delta=\sqrt{\frac{\mu L}{u_0\rho}}. \qquad (39)$$

The introduction of (39) into (38) leads to dimensionless equation

$$\frac{\partial U}{\partial X}+\frac{v_0L}{u_0\delta}\frac{\partial V}{\partial Y}=0, \qquad (40)$$

where the continuity of the flow is retained at the following ratio between the characteristic scales of the flow:

$$\frac{v_0L}{u_0\delta}=1, \quad v_0=\frac{\delta}{L}u_0=u_0\sqrt{\frac{\upsilon}{u_0L}}. \qquad (41)$$

Let us suppose that the Marangoni effect is a result of temperature gradient at the interface. In this case the characteristic velocity of the Marangoni effect can be determined from the equation [1]:

$$\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}=\frac{\partial\sigma}{\partial x}=\frac{\partial\sigma}{\partial t}\frac{\partial t}{\partial x}. \qquad (42)$$

If introducing (39) in (42) (and temperature scale Δt) we reach the condition for the existence of Marangoni effect and its characteristic velocity:

$$u_0=\frac{\delta}{L}\frac{\Delta t}{\mu}\frac{\partial\sigma}{\partial t}. \qquad (43)$$

The introduction of (43) in (41) allows the determination of the characteristic velocity of the secondary flow liable for the increase of the mass transfer rate as a result of the Marangoni effect:

$$v_0=\frac{\Delta t}{\rho u_0L}\frac{\partial\sigma}{\partial t}. \qquad (44)$$

In the case of absorption of CO_2 in H_2O and temperature change because of chemical reaction, the order of the velocity v_0 can be determined:

$$v_0\sim 10^{-8}m/s. \qquad (45)$$

In cases of non-linear mass transfer, characteristic scales have to be introduced into (34):

$$v = v_0 V, \quad c = \Delta c C, \quad y = \delta_c Y, \quad \delta_c = \sqrt{\frac{\delta l}{u_0}}. \qquad (46)$$

From (34) and (46) directly can be obtained the condition for an existence of the non-linear mass transfer effect and its characteristic velocity:

$$v_0 = \frac{M \Delta c}{\rho_0^*} \sqrt{\frac{u_0 D}{L}}. \qquad (47)$$

The v_0 order of the velocity in (47) at analogous conditions with (45) is determined directly:

$$v_0 \sim 10^{-6} m/s. \qquad (48)$$

The obtained results (45) and (48) show, that in systems with an intensive interphase mass transfer the non-linear effects are related to the concentration gradients normally oriented to the interface, and are not related to the temperature gradients at the phase boundary. This proportion between the effect of non-linear mass transfer and the Marangoni effect is based mainly on the following three reasons:

the normal component of the velocity v_0 is always smaller than the induced tangential component (see (41));

to the large concentration gradient Δc correspond small temperature gradients Δt because of the small heat effect of the absorption;

the absence of surface-active substances.

The non-linear mass transfer effect and the Marangoni effect can determine the mass transfer rate not only by additional convective flows, but also by loss of stability. In these cases accidental disturbances lead to self-organized dissipative structures with very intensive mass transfer [1]. The stability of these systems depends mainly from v_0, which is the reason the process to be limited again from the non-linear mass transfer.

Conclusion

The presented theoretical analysis shows that the effects of the Stephan flow and the non-linear mass transfer at large concentration gradients have different physical nature, and as a result different mathematical models.

The Stephan flow [12] appears at heterogenic chemical reactions (with changing volume of the reaction mixture) or liquid–vapour phase transition at interface (boiling, condensation).

The non-linear mass transfer [1] appears at interphase mass transfer in gas (liquid) – solid, gas-liquid and liquid-liquid systems as a result of large concentration gradients.

The Marangoni effect does not appear in gas-liquid and liquid-liquid systems [7,8] at absence of surface active agents, because of the small heat effect of the dissolution process.

REFERENCES

1. BOYADJIEV C. B., BABAK V. N.: Non-Linear Mass Transfer and Hydrodynamic Stability, Elsevier, Amsterdam, pp. 500, 2000
2. BOYADJIEV C. B.: Hung. J. Ind. Chem.,*1996*, 24, 35-39
3. BOYADJIEV C. B., HALATCHEV I.: Int. J. Heat Mass Transfer, *1998*, 41, 939-944
4. BOYADJIEV C. B.: Int. J. Heat Mass Transfer, *2000*, 43, 2749-2757
5. BOYADJIEV C. B.: Int. J. Heat Mass Transfer, *2000,* 43, 2759-2766
6. KRYLOV V. S., BOYADJIEV C. B.: Non-Linear Mass Transfer, Institute of Thermophysics, Novosibirsk, pp.231, 1996, (in Russian).
7. BOYADJIEV C. B., HALATCHEV I.: Int. J. Heat Mass Transfer, *1998*, 41, 197-202
8. BOYADJIEV C. B., DOICHINOVA M., Hung. J. Ind. Chem., *1999*, 27, 215-219
9. BOYADJIEV C. B.: Hung. J. Ind. Chem., *1998*, 26, 181-187
10. SAPUNDZHIEV T., BOYADJIEV C. B.: Russian J. Eng. Thermophysics, *1993*, 3, 185-198
11. FRANK-KAMENETSKII D. A.: Diffusion and Heat Transfer in Chemical Kinetics, Science, Moscow, pp.491, 1967
12. BIRD R. B., STEWART W. E., LIGHTFOOT E. N.: Transport Phenomena, J. Wiley, New York, pp.687, 1965