

COMPUTER DRAWING OF ENTHALPY-HUMIDITY CHARTS

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Enthalpy-humidity charts for a gas-vapour mixture still provide a fast and easy tool for use in process calculations in drying, solvent recovery and related processes. They have to be individually constructed for each new system encountered. The computer graph-plotting method provides a fast and reliable way of enthalpy-humidity chart production.

Theoretical principles of computerized *i*-*Y* chart plotting are provided. Exemplary charts for systems of nitrogen and benzene, *n*-heptane, and cyclohexane are given.

Introduction

Traditionally parameters of a humid gas were presented on a psychrometric chart made in rectangular linear coordinates: temperature *v.s.* absolute humidity *Y*. Such charts containing lines of constant relative humidity, and constant wet-bulb temperature are still in use in psychrometry, *i.e.* humidity measuring by wet and dry-bulb temperature read-outs.

In 1923, R. MOLLIER [1] introduced an enthalpy-humidity chart (*i*-*Y* chart) allowing for simultaneous humidity, temperature and enthalpy read-outs. The chart was initially constructed in rectangular enthalpy-humidity coordinates. RAMZIN modified the chart by introducing oblique axes, which allow for better resolution of RH lines in low humidity area.

Figure 1 presents a comparison of both types of charts, *i.e.* psychrometric versus enthalpy-humidity chart.

The relationships on which both charts are made can be directly used in a computer programme [2], however, the graphical form of a chart is handy in practical use and visually appealing when used for presentation of the computed results.

There are many industrial processes (*e.g.* drying) in which mixtures of vapour and inert gas other than water and air are dealt with. Enthalpy-humidity charts for such systems are only occasionally encountered in literature (*e.g.* [3]—trichloroethylene-air and [4]— N_2 -methanol; N_2 -toluene). On the other hand, a fully computerized production of enthalpy-humidity charts

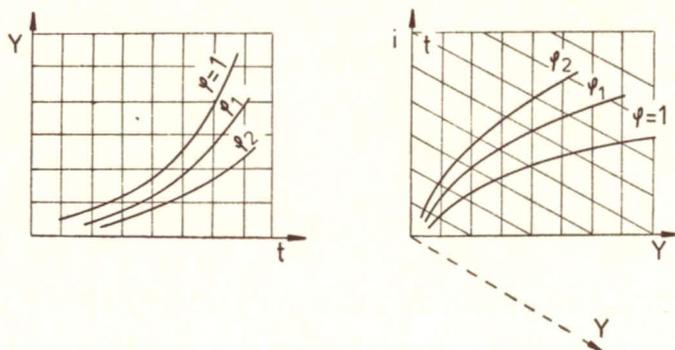


Fig. 1

Two types of charts for humid gases
 a — psychrometric, b — enthalpy-humidity (Mollier chart)

is feasible and has already been implemented by the leading drier manufacturers [5]. This paper presents the principles of the computerized drawing of i - Y charts, together with exemplary charts produced in this way.

Summary of Thermodynamic Relationships for a Gas-Vapour Mixture

Absolute humidity Y is a number of kilograms of vapour per kilogram of dry gas. *Relative humidity* φ is a ratio of actual partial pressure of vapour to that of saturated vapour at the same temperature:

$$\varphi = \frac{p}{p_s} \quad (1)$$

On the basis of DALTON'S law, there is a following relationship between the two humidities:

$$Y = D \frac{\varphi p_s}{P - \varphi p_s} \quad (2)$$

where $D = M_A/M_B$. (3)

Dew point temperature t_D is the temperature at which the vapour in gas becomes saturated. By cooling gas below this temperature, fog is formed.

Enthalpy of humid gas i is calculated additively according to the simplified formula:

$$i = (\bar{c}_B + \bar{c}_A Y)t + \Delta h_{v0} Y \quad (4)$$

where specific heats \bar{c}_B and \bar{c}_A are averaged values.

Averaging is performed between the reference temperature and temperature t . For substances whose melting point is below 0 °C, the reference temperature is taken as 0 °C. The latent heat of vaporization Δh_{v0} is taken at the same temperature.

Temperature of adiabatic saturation t_{as} corresponds to the equilibrium between gas and liquid, and can be calculated according to the following equation:

$$\frac{i - i_{as}^*}{Y - Y_{as}^*} = -(\bar{c}_A l)_{as} t_{as} \tag{5}$$

Wet bulb temperature t_{wb} is reached by a small amount of liquid exposed to a large quantity of gas. Contrary to the t_{as} it is controlled by kinetics of heat and mass transfer, and therefore depends on the Lewis number $Le = Sc/Pr$. Theoretical (thermodynamical) wet bulb temperature can be calculated from the following formula [4]:

$$\frac{t - t_{wb}}{Y - Y_{wb}^*} = -\beta Le^{-2/3} \frac{D}{D + Y_{wb}^*} (1 + Y_{wb}^*) \frac{(\Delta h_v)_{wb}}{c_H} \tag{6}$$

where:

$$c_H = \bar{c}_A Y + \bar{c}_B \tag{7}$$

The coefficient β is a correcting factor usually close to 1.

Geometry of the Enthalpy-Humidity Chart

An enthalpy-humidity chart is made in an oblique coordinate system. The vertical axis contains an enthalpy scale and the oblique axis an absolute humidity scale. Both scales are linear. In practice the projection of the oblique Y axis on a horizontal one is used, since the range of parameters between these two is of little interest in industrial applications. Finally on a rectangular chart we have a grid of vertical constant Y lines and oblique constant i lines (isoenthalps). The angle between them is arbitrarily chosen as 135° for an air-water system, although it can be selected otherwise. As each point in the chart simultaneously gives enthalpy, humidity and temperature, a grid of isotherms is also placed in the chart. On top of that, lines of constant relative humidity φ are superimposed with the most important one for $\varphi = 100\%$ — the saturation line.

For further information, lines of constant adiabatic saturation temperature and constant wet bulb temperature are added. All these elements of i - Y chart geometry are shown in *Fig. 2*.

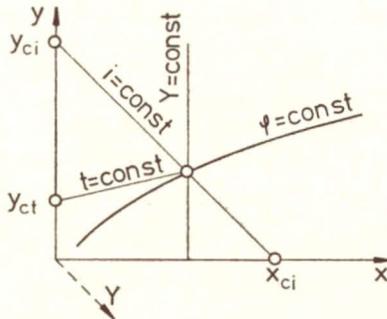


Fig. 2
Elements of enthalpy-humidity chart geometry

When using a computerized graph-plotter, all these lines must be converted into x, y coordinates of the plotter. Therefore equations of all basic lines of i - Y chart must be first derived. For that purpose we assume that at the origin of graph-plotter coordinates ($x=0$ and $y=0$) we also have $i=0$ and $Y=0$. Additionally, scaling factors for enthalpy and humidity scales s_i and s_Y are set in such a way that:

$$i = s_i l \quad (8)$$

$$Y = s_Y l \quad (9)$$

where l is a certain distance from the origin in units of the plotter (mm or inches).

Isoenthalp Equation

Isoenthalp as a straight line has the following equation:

$$y = a_1 x + y_{ci} \quad (10)$$

The intercept y_{ci} on the basis of (8) is given by:

$$y_{ci} = i/s_i \quad (11)$$

The slope a_1 can be calculated from both intercepts x_{ci} and y_{ci} . Usually the slope is selected in such a way that:

$$y_{ci}s_i = \Delta h_{v0} x_{ci} s_Y \quad (12)$$

It is equivalent with an assumption that $t=0^\circ$ isotherm coincides with the horizontal axis x (as will be proven later). Some authors select other forms of condition (12), which leads to a different location of the isotherm 0°C .

Combining (10), (11) and (12) we have:

$$a_1 = -\Delta h_{v0}(s_Y/s_i) \quad (13)$$

and therefore the isoenthalp equation has the following form:

$$y = -\Delta h_{v0}(s_Y/s_i)x + i/s_i \quad (14)$$

Constant Y Line Equation

As vertical lines they have a trivial form of:

$$s_Y x = \text{const} \quad (15)$$

Isotherm Equation

Local slope a_t of an isotherm is given by the ratio $\Delta y/\Delta x$ as can be seen in Fig. 3. The value of Δy can be found from isoenthalps as:

$$\Delta y = a_1 \Delta x + \Delta i/s_i \quad (16)$$

On the other hand, from (14) it follows that:

$$\Delta i = \bar{c}_A t \Delta Y + \Delta h_{v0} \Delta Y \quad (17)$$

hence:

$$a_t = \frac{\Delta y}{\Delta x} = \frac{a_1 \Delta x + (\bar{c}_A t s_Y \Delta x + \Delta h_{v0} s_Y \Delta x)/s_i}{\Delta x} = a_1 + (\bar{c}_A t + \Delta h_{v0}) s_Y/s_i \quad (18)$$

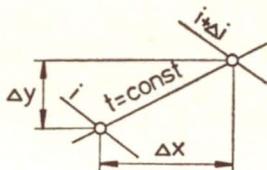


Fig. 3

Auxiliary diagram for evaluation of slope of isotherm

which on the basis of (13) is equal to:

$$\alpha_t = \bar{c}_A t s_Y / s_1 \quad (19)$$

As can easily be seen from (19) the slope is constant for given t (but changes with temperature). Therefore, the isotherms are straight lines.

The full isotherm equation has the form:

$$y = \alpha_t x + y_{ct} \quad (20)$$

where y_{ct} can be easily found from the enthalpy scale (on the vertical axis $Y=0$) as:

$$y_{ct} = \frac{\bar{c}_B t}{s_1} \quad (21)$$

Finally we have:

$$y = \bar{c}_A t \frac{s_Y}{s_1} x + \bar{c}_B t / s_1 \quad (22)$$

It is worth noticing that for $t=0$ $y \equiv 0$ so the isotherm 0°C is identical with the x axis.

In the fog area, isotherms follow a different equation (that of an adiabatic saturation line), which is explained in most textbooks. This area is of little industrial interest and will be omitted here.

Other Lines

The remaining lines are not straight (in principle it also concerns t_{as} and t_{wb} lines). Deriving a detailed equation for these lines is not always simple or feasible, taking into consideration changes of specific heats and other parameters with temperature. Other plotting procedures are then employed.

The scale factors s_Y and s_1 are selected on the basis of the required dimensions of the chart A_x and A_y (in graph-plotter units) and maximum values of temperature and humidity to be covered by the chart. They are given by the formulas:

$$s_Y = \frac{Y_{\max}}{A_x} \quad s_1 = \frac{\bar{c}_B t_{\max}}{A_y} \quad (23, 24)$$

Plotting of the individual elements of the enthalpy-humidity charts is performed by procedures described below.

Procedure: ISOENTHALP

Draws subsequent isoenthalp for a given increment Δi . Line drawing subroutine of graph plotter is used. Each time end points of an isoenthalp are calculated prior to plotting. The end points correspond to an intersection of the isoenthalp and chart frame.

Procedure: ISOHUMID-Y

Using the line plotting subroutine of graph-plotter plots vertical lines for a given increment ΔY .

Procedure: ISOTHERM

Using the line plotting subroutine plots isotherms for a given increment Δt . The left end of each isotherm corresponds to the left side of the frame, the right end corresponds to the saturation line or the frame side (right or upper). The intersection point of an isotherm with saturation line is computed using Eq. (2). Additionally a boiling point isotherm (dashed line in *Figs. 4-6*) is drawn.

Procedure: ISOHUMID-RH

Draws $\varphi = \text{const}$ lines for given values of φ starting with $\varphi = 1$. They are indicated on the right-hand side of the chart frame. It employs a three-point parabole fitting procedure of the plotter.

For a given φ the following procedure is performed:

- three values of temperature are taken (starting with $t = 0$)
- corresponding values of Y are computed from Eq. (2)
- coordinates of these three points are converted into the x, y coordinates of the plotter
- a curve is drawn through these points and the procedure repeated with the next three values of t until $t = t_b$ or until $Y > Y_{\text{max}}$.

Procedure: AST LINE

A function plotting subroutine of the plotter is used in general to accommodate changes of specific heats with temperature. If high accuracy can be sacrificed, then a straight line procedure may be used.

Procedure: WBT LINE

Because of the implicit form of the relationship (6), a similar procedure as for relative humidity lines was employed. For a given t_{wb} three subsequent values of t are taken, starting with a point on the saturation line. Then by a numerical method Eq. (6) is solved to find corresponding values of Y . After conversion of coordinates, a curve is drawn through these points and new values of temperature taken. This procedure was not employed in *Figs. 4-6*.

MOLLIER CHART FOR BENZENE-NITROGEN
PRESSURE 101325.0 PASCALS

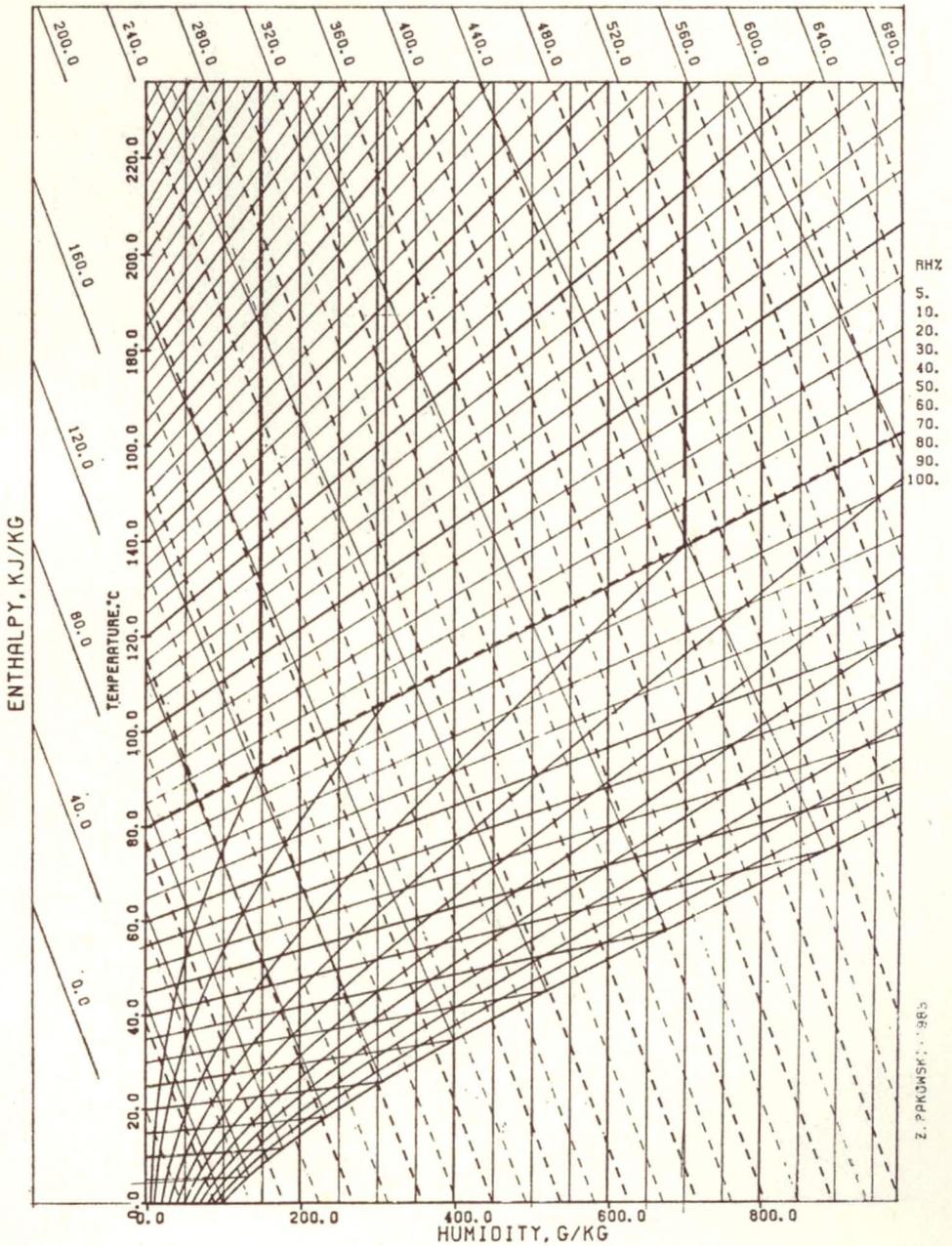


Fig. 4
Enthalpy-humidity chart for nitrogen-benzene

MOLLIER CHART FOR N-HEPTANE-NITROGEN
PRESSURE 101325.0 PASCALS

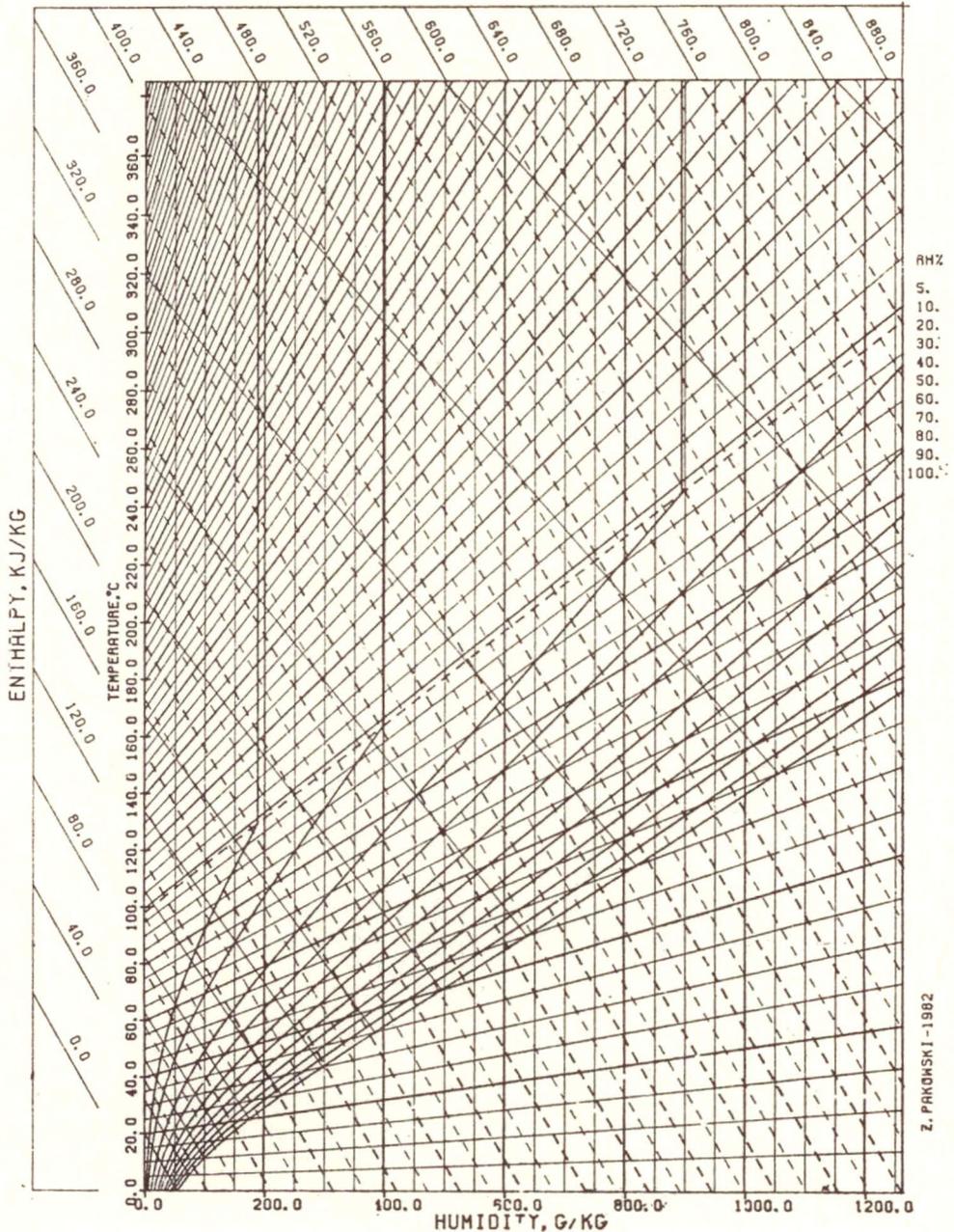


Fig. 5
Enthalpy-humidity chart for nitrogen-n-heptane

MOLLIER CHART FOR CYCLOHEXANE-NITROGEN
PRESSURE 101325.0 PASCALS

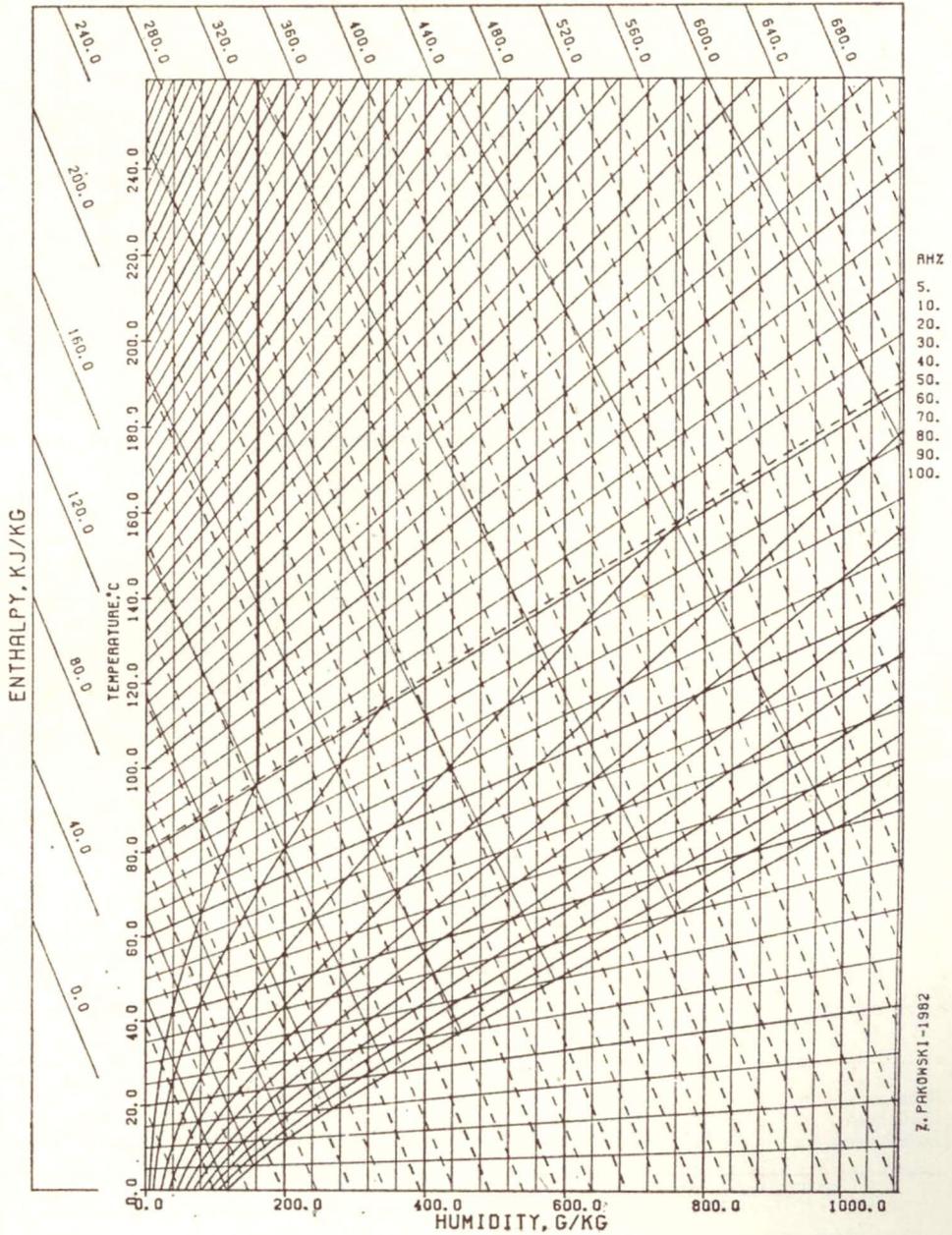


Fig. 6
Enthalpy- humidity chart for nitrogen-cyclohexane

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Auxiliary procedures

These procedures draw picture frame, temperature, enthalpy and humidity scales and print the chart's title.

Closure

The above described procedures were used in the programme GRAFITE written in FORTRAN for a ZETA 3653 SC plotter. A series of charts for most common systems were drawn [6]. *Figures 4-6* contain three charts for selected systems. Data used for plotting these charts are given in the Appendix. The method is highly efficient and can be used for chart production in the design of closed cycle dryers for pharmaceutical and other industries.

Appendix**Data Used for Plotting the Enclosed Charts**

1. Specific heat of gas and vapour was calculated from the following equation:

$$cl = a + b \times 10^{-2}T + c \times 10^{-6}T^2 + d \times 10^{-9}T^3$$

with coefficients:

	a	b	c	d
benzene	-0.4716	6.2177	-4.079	1.013
n-heptane	0.82485	6.0392	-2.739	0.4144
cyclohexane	-0.7871	8.0170	-4.4361	0.91344
nitrogen	1.05679	-0.1973	0.4948	-0.18834

2. Specific heat of liquid was calculated from the following equation:

$$cl = a + b \times 10^{-2}T + c \times 10^{-4}T^2 + d \times 10^{-7}T^3$$

with coefficients:

	a	b	c	d
benzene	-6.2007	6.4728	-1.8296	1.8459
n-heptane	-0.12796	1.651	-0.4262	0.46061
cyclohexane	-5.3759	5.6061	-1.4695	1.3510

3. Saturated vapour pressure was calculated from the following equation:

$$\log_{10} (p_s/133.322) = a - b/(c+t)$$

with coefficients:

	a	b	c
benzene	6.89272	1203.531	219.888
n-heptane	6.90253	1267.828	216.823
cyclohexane	6.84941	1206.001	223.148

4. Molar masses and latent heat of vaporization at 0 °C were as follows:

	M	Δh_{v0}
benzene	78.11	446
n-heptane	100.20	381
cyclohexane	86.17	406
nitrogen	28.01	—

SYMBOLS

- A_x — dimension of chart in x-direction, mm
 A_y — dimension of chart in y-direction, mm
 a_1 — slope of the isoenthalp, —
 a_t — slope of the isotherm, —
 c_A — specific heat of vapour (moisture), kJ/kgK
 c_B — specific heat of gas (inert), kJ/kgK
 $c_{A'l}$ — specific heat of liquid (moisture), kJ/kgK
 c_H — humid heat — Eq. (7), kJ/kgK
 D — molar mass ratio, —
 Δh_v — latent heat of vaporization, kJ/kg
 Δh_{v0} — latent heat of vaporization at standard temperature (0 °C), kJ/kg
 i — enthalpy of humid gas, kJ/kg dry gas
 Le — LEVIS number, —
 l — distance from the origin of chart, mm
 M_A — molar mass of moisture, kmol/kg
 M_B — molar mass of gas, kmol/kg
 P — ambient pressure, Pa
 p — vapour pressure, Pa
 p_s — saturated vapour pressure, Pa
 s_1 — scaling factor for enthalpy, kJ/kg mm
 s_Y — scaling factor for humidity, kg/kg mm
 T — temperature, K
 t — temperature, °C
 t_{max} — maximum temperature on the chart, °C
 x — horizontal coordinate of graph-plotter, mm
 Y — absolute humidity, kg/kg dry gas
 Y_{max} — maximum absolute humidity on the chart, kg/kg dry gas
 y — vertical coordinate of graph-plotter, mm
 β — correction factor in Eq. (6), —
 φ — relative humidity (RH), —

Subscripts and superscripts

- as — adiabatic saturation point
- wb — wet bulb point
- * — saturated
- — averaged between standard and given temperature

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РЕЗЮМЕ

Диаграммы типа энтальпия-влажность газовой-паровой смеси оказываются весьма полезным инструментом в расчетах процессов сушки и извлечения растворителей. Эти диаграммы должны быть составлены в соответствии с изученной системой в каждом конкретном случае. Вырисовывание диаграмм энтальпия-влажность с помощью ЭВМ является быстрым и надежным методом.

В настоящей статье обсуждены теоретические предпосылки составления диаграмм $i-Y$. В качестве примеров проведены диаграммы систем азот-бензол, -гептан -циклогексан полученные на ЭВМ.