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STUDIES ON THE HYDRODINAMICS OF FLUIDIZED LAYERS II. STREAMING OF FLUID, PARTICLE MOTION AND LAYER EXPANSION IN SYSTEMS FLUIDIZED WITH A LIQUID

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Starting from the physical model, basic equations are derived for the flow of the fluid, particle motion and changes in particle density along the radius. These equations enable the fluid-mechanical properties of fluidized systems to be described.

The basic equations derived are applied to systems fluidized with a liquid, and equations are presented for the calculation of the inter-granular liquid flow rate, of the change in grain flow rate along the radius and of the void fraction.

INTRODUCTION

Various theories have been described in literature for the description of the fluid mechanical properties, such as expansion and viscosity of the layer, and the motion of particles, etc., in fluidized layers [1, 2, 3, 4, 5, 6] etc. Based on these theories a number of equations was derived;however, the practical application of these is cumbersome and difficult.

In the following, based on a physical model, the derivation of equations enabling the calculation of the most important fluid mechanical parameters will be presented. Due to the compositeness of the system and the mathematical difficulties encountered as a consequence of the complicated connections, a large number of approximations and neglections had to be applied and consequently the final conclusions can be regarded only as semi-empirical formulas. However, it was not considered the aim of the present work to elaborated an exact theory - this being, after all, in the opinion of the authors quite impossible due to the complexity and precariousness of the system - but to arrive at connections which enable the fluid mechanical parameters, necessary for design and optimalization, to be calculated to an adequate degree of accuracy.

In order to present a fluid mechanical description of fluidized systems, the physical model illustrated in Fig.1 was taken as a starting point and the following assumptions and restrictions were applied.

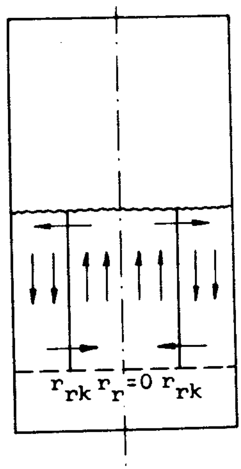

Fig.1

a) The particles in the layer are sphere-shaped; however, they are not necessarily of a uniform size.

b) The streaming of the medium is described by the <u>Navier-Stokes</u> equation. According to this, a rate gradient is built up along the radius.

c) The flow rate of the medium in the direction y is constant, but the velocity of the particles at a point y = 0 is zero and increases with increasing y value. This assumption is not wholly compatible with the uniform flow rate of the medium in the direction y. However, this assumption is justified by the good agreement between data calculated by the obtained formulas and determined experimentally. This will be reported in the next paper of this series.

d) Motion of the particles along the radius is brought about by the dynamic equilibrium of two forces. The rate profile in the direction of and decreasing with the radius tends to force the particles in the direction of increasing radius. It can be shown that - were there no force of opposite direction - a minimum resistance of the system would be obtained in a extreme state, where all the particles would adhere to the wall of the apparatus and the medium would stream in the space enclosed by the annular cluster of particles. The unoriented motion of the particles, brought about by random fluctuations in the flow rate of the medium, acts against this force of centrifugal nature. If the particles were of uniform size, these fluctuations would cause collisions more rarely and would manifest themselves in a wave-like expansion or contraction. On the other hand, if the size of the particles is not identical, random fluctuations will cause frequent collisions and the particles will be forced - in a manner similar to the process of gas diffusion - in the direction of lower particle density, i.e. in a centripetal direction. In steady state, the radial density distribution of the particles is determined by the equilibrium of these two forces.

In order to present a fluid mechanical description of a fluidized layer, it is necessary to known the values of radial changes in the flow rate of the fluid streaming between the particles, the particle flow rate, and the void fraction.

FLOW OF THE FLUID

In order to describe the flow of the fluid, the Navier-Stokes equations, written for the case of cylindrical co-ordinates, can be used. The equations are the following [7]:

$$\rho' \left(\frac{\partial u'_R}{\partial \tau} + u'_R \frac{\partial u'_R}{\partial r} + \frac{u'_\varphi}{r} \frac{\partial u'_R}{\partial \varphi} + \frac{u'_\varphi}{r} + u'_y \frac{\partial u'_R}{\partial y} \right) =$$

$$= \frac{\partial p}{\partial r} + \mu' \left\{ \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial (r u'_R)}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2 u'_R}{\partial \varphi^2} - \frac{2}{r^2} \frac{\partial u'_R}{\partial \varphi} + \frac{\partial^2 u'_R}{\partial y^2} \right\} + P_R \qquad (1)$$

$$\rho' \left(\frac{\partial u'_\varphi}{\partial \tau} + u'_R \frac{\partial u'_\varphi}{\partial r} + \frac{u'_\varphi}{r} \frac{\partial u'_\varphi}{\partial \varphi} + \frac{u'_R u'_\varphi}{r} + u'_y \frac{\partial u'_\varphi}{\partial y} \right) =$$

$$= - \frac{1}{r} \frac{\partial p}{\partial \varphi} + \mu' \left\{ \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial (r u'_\varphi)}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2 u'_\varphi}{\partial \varphi^2} + \frac{2}{r^2} \frac{\partial u'_\varphi}{\partial \varphi} + \frac{\partial^2 u'_\varphi}{\partial y^2} \right\} + P_\varphi \qquad (2)$$

$$\rho' \left(\frac{\partial u'_y}{\partial \tau} + u'_R \frac{\partial u'_y}{\partial r} + \frac{u'_\varphi}{r} \frac{\partial u'_y}{\partial \varphi} + u'_y \frac{\partial u'_y}{\partial y} \right) =$$

$$= - \frac{\partial p}{\partial y} + \mu' \left[\frac{1}{r} \frac{\partial \left(r \frac{\partial u'_y}{\partial r}\right)}{\partial r} + \frac{1'}{r^2} \frac{\partial^2 u'_y}{\partial \varphi^2} + \frac{\partial^2 u'_y}{\partial y^2} \right] + P_y \qquad (3)$$

There is no acceleration at a given point of the layer and accordingly we can write:

$$\frac{\partial u'_R}{\partial \tau} \equiv \frac{\partial u'_\varphi}{\partial \tau} \equiv \frac{\partial u'_y}{\partial \tau} \equiv 0 \qquad (4)$$

The pressure does not alter with changing r and φ, and consequently

$$\frac{\partial p}{\partial r} \equiv \frac{\partial p}{\partial \varphi} \equiv 0 \qquad (5)$$

The system is of cylindrical symmetry and so we obtain:

$$\frac{\partial u'_R}{\partial \varphi} \equiv \frac{\partial^2 u'_R}{\partial \varphi^2} \equiv \frac{\partial u'_\varphi}{\partial \varphi} \equiv \frac{\partial^2 u'_\varphi}{\partial \varphi^2} \equiv \frac{\partial u'_y}{\partial \varphi} \equiv \frac{\partial^2 u'_\varphi}{\partial \varphi^2} \equiv 0 \qquad (6)$$

There is no force acting upon the system in the direction r and φ:

$$P_R \equiv P_\varphi \equiv 0 \qquad (7)$$

It is assumed that the system does not flow in the directions r and φ:

$$u'_R \equiv u'_\varphi \equiv \frac{\partial u'_R}{\partial y} \equiv \frac{\partial u'_R}{\partial r} \equiv \frac{\partial^2 u'_R}{\partial y^2} \equiv \frac{\partial u'_\varphi}{\partial y} \equiv \frac{\partial u'_\varphi}{\partial r} \equiv \frac{\partial^2 u'_\varphi}{\partial y^2} \equiv 0 \tag{8}$$

It follows from assumption (8) that:

$$\frac{\partial u'_y}{\partial y} \equiv \frac{\partial^2 u'_y}{\partial y^2} \equiv 0 \tag{9}$$

From Equation (1), (2) and (3), considering Equations (4), (5), (6), (7), (8) and (9), we obtain:

$$\frac{dp}{dy} \equiv \mu' \frac{1}{r} \frac{d}{dr} \left(r \frac{du'_y}{dr}\right) + P_y \tag{10}$$

The pressure in the layer is proportional to the weight of the layer [8]:

$$\Delta p = A_1 \left[\bar{\varepsilon}(\rho - \rho') + \bar{\varepsilon}'\rho'\right] gy \tag{11}$$

from which we obtain:

$$\frac{dp}{dy} = A_1 g \left[\bar{\varepsilon}(\rho - \rho') + \bar{\varepsilon}'\rho'\right] \tag{12}$$

The force acting upon the unit volume is equal to the sum of the force acting upon the particles plus that acting upon the fluid.

The force acting upon one particle is, according to Stokes, proportional to the difference in the velocities of the particle and the fluid [9]:

$$P = a_1(u'_y - u_y) \tag{13}$$

If the difference is equal to the velocity of the free fall of the particle, it is self-ecident that the force is equal to the Archimedean weight of the particle and we can write:

$$vg(\rho - \rho') = a_1 u_e \tag{14}$$

The force acting upon the particles present in the unit volume is, as derived from Equations (13) and (14):

$$P = \frac{vg(\rho - \rho')}{u_e} (u'_y - u_y) \tag{15}$$

The force acting upon the fluid present in the unit volume is:

$$P' = \varepsilon'\rho' g \tag{16}$$

The sum of the forces acting upon the particles and the fluid present in the unit volume is, on the basis of Equations (15) and (16), further Equation (5), the following:

$$P_y = \frac{g(\rho - \rho')\varepsilon}{u_e} (u'_y - u_y) + \varepsilon'\rho' g \tag{17}$$

The flow of the fluid can be described, on the basis of Equations (10), (12) and (17), by the following formula:

$$A_1[\bar{\varepsilon}(\rho - \rho') + \bar{\varepsilon}'\rho']g =$$

$$= \mu' \frac{1}{r} \frac{d}{dr} \left(r \frac{du'_y}{dr}\right) + \frac{g\varepsilon(\rho - \rho')}{u_e} (u'_y - u_y) + \varepsilon'\rho' g \tag{18}$$

Motion of the Particles

In the following, the equation describing the motion of the particles will be presented. On the basis of the second Newton's Law, and taking Equation (15) into consideration, we can

$$v\rho \frac{du_y}{d\tau} = \frac{v(\rho - \rho')g}{u_e} (u'_y - u_y) - v(\rho - \rho')g \tag{19}$$

After rearrangement and introduction of relative variables we obtain:

$$\frac{du_{ry}}{d\tau} = \frac{(\rho - \rho')g}{\rho u_e} (u'_{ry} - u_{ry} - 1) \tag{20}$$

The starting condition is:

$$u_{ry}(0) = 0 \tag{21}$$

The solution of Equation (20), taking starting condition - Eq. (21) - into consideration, is:

$$u_{ry} = (u'_{ry} - 1)\left\{1 - \exp\left[-\frac{(\rho - \rho')g\tau}{\rho u_e}\right]\right\} \tag{22}$$

Velocity can be written as the differential quotient of displacement according to time:

$$\frac{dy}{d\tau} = u_e(u'_{ry} - 1)\left\{1 - \exp\left[-\frac{(\rho - \rho')g\tau}{\rho u_e}\right]\right\} \tag{23}$$

The starting condition is:

$$u(0) = 0 \tag{24}$$

The solution of the differential equation, considering starting condition - Equation (24) - is the following:

$$y = (u'_{ry} - 1)u_e\tau - \frac{(u'_{ry} - 1)u_e^2\rho}{(\rho - \rho')g}\left\{1 - \exp\left[-\frac{(\rho - \rho')g\tau}{\rho u_e}\right]\right\} \tag{25}$$

Rearranging Equation (25):

$$y = (u'_{ry} - 1)u_e\tau\left\{\left[1 - \frac{u_e\rho}{(\rho - \rho')g\tau}\right]\left\{1 - \exp\left[-\frac{(\rho - \rho')g\tau}{\rho u_e}\right]\right\}\right\} \tag{26}$$

Let us approximate the exponential expression by the first two members of the series, i.e.:

$$\exp\left[-\frac{(\rho - \rho')g\tau}{\rho u_e}\right] \approx \frac{1}{1 + \frac{(\rho - \rho')g\tau}{\rho u_e}} \tag{27}$$

It was found that a comparative test made with the degree of approximation - Equation (27) - showed the approximation to be better if a coefficient of 0.85 is used. Accordingly, from Equations (26) and (27) we have:

$$y = 0.85(u'_{ry} - 1)u_e\tau \frac{\frac{(\rho - \rho')g\tau}{\rho u_e}}{1 + \frac{(\rho - \rho')g\tau}{\rho u_e}} \tag{28}$$

By solving the Equation (28) for τ and introducing the dimensionless quantity:

$$A_4 = \frac{g(\rho - \rho')Y_m\varepsilon_m}{\rho u_e^2} \tag{29}$$

we arrive at:

$$2\tau = \frac{y}{0.85\ u_e(u'_{ry} - 1)}\left[1 + \sqrt{1 + \frac{3.4\ \bar{\varepsilon}(u'_{ry} - 1)}{A_4}}\right] \tag{30}$$

The mean velocity of the particles along the height is:

$$\bar{u}_y = \frac{y}{\tau} \tag{31}$$

The mean relative velocity of the particles along the height, as determined from Equations (30) and (31), is

$$\bar{u}_{ry} = \frac{1.7(u'_{ry} - 1)}{1 + \sqrt{1 + \frac{3.4\ \bar{\varepsilon}(u'_{ry} - 1)}{A_4}}} \tag{32}$$

Changes in Particle Density Along the Radius

If a difference in particle density is built up in the fluidized layer along the radius, a streaming of the particles along the radius will start in consequence of this difference.

This equalization process can start only if the particles perform an oscillating motion, since in the case of stationary particles the difference in particle density is not a sufficient cause for the motion. If the particles are floating, their oscillating motion can be observed even by the naked eye. The cause of this phenomenon is - in all probability - the fluctuation of the fluid flow rate in time. The latter has been measured by a number of researchers, e.g. by AEROV [10] among others. The Equation (3) serving the fluid mechanical description of fluidized layers can be derived on the basis of the foregoing.

The volume of the particles moving during unit time from one granular layer into another granular layer in a cylinder jacket of the radius r and the height dy is proportional to the surface area of the particles present at the cylinder jacket ($2\pi r\varepsilon dy$) and to the vibration rate. A fraction of the arriving particles corresponding to the void fraction (ε') enters the neighbouring layer at a distance Y_v, and accordingly the volume of the arriving particles (dv_I) is:

$$dv_I = 2\pi\varepsilon r u_v dy \tag{33}$$

The volume of the particles returning from the neighbouring layer (dv_{II}), if the void fraction was decreased by $\Delta\varepsilon$, while r was increased by Y_v, is the following:

$$dv_{II} = 2\pi\varepsilon u_v dy(r + Y_v)(\varepsilon' - \Delta\varepsilon') \tag{34}$$

The resultant volume of the transient particles (dv_{III}) is, evidently, the difference of the two volumes:

$$dv_{III} = dv_I - dv_{II} \tag{35}$$

and consequently from Equations (33), (34) and (35) we obtain:

$$dv_{III} = 2\pi u_v \varepsilon dy(r\Delta\varepsilon' - Y_v\varepsilon' + Y_v\Delta\varepsilon') \tag{36}$$

The expression $Y_v \Delta\varepsilon'$ is a secondary small value and may be neglected:

$$\Delta\varepsilon' Y_v \approx 0 \tag{37}$$

By approximating the $\varepsilon'(r)$ function by a continuous function:

$$\Delta\varepsilon' \approx Y_v \frac{d\varepsilon'}{dr} \tag{38}$$

From Equations (36), (37) and (38) we obtain:

$$dv_{III} = 2\pi u_v Y_v \varepsilon dy \left(r \frac{d\varepsilon'}{dr} - \varepsilon'\right) \tag{39}$$

The particle volume that passed during unit time can also be expressed by the velocity of the particles along the radius:

$$dv_{III} = 2\pi\varepsilon u_R r dy \tag{40}$$

The particle velocity along the radius is, according to Equations (39) and (40), the following:

$$u_R = - u_v Y_v \left(\frac{\varepsilon'}{r} - \frac{d\varepsilon'}{dr}\right) \tag{41}$$

The distance between the particles may be regarded as equal to the vibrational length of the particles; the product of the latter and the vibration rate may be termed the vibration coefficient:

$$K_v = u_v Y_v \tag{42}$$

The continuity rule is:

$$\frac{\partial u_R}{\partial r} + \frac{u_R}{r} + \frac{\partial u_y}{\partial y} = 0 \tag{43}$$

From Equations (41), (42) and (43) we obtain:

$$K_v \frac{d^2\varepsilon}{dr^2} = - \frac{\partial u_y}{\partial y} \tag{44}$$

Let us take the integral means according to height:

$$\frac{1}{Y} \int_o^Y \frac{\partial u_y}{\partial y} \cdot dy = \frac{1}{Y} [(u_y)_{y=Y} - (u_y)_{y=0}] \tag{45}$$

As a first approximation, arithmetical means may be taken:

$$\bar{u}_y = \frac{(u_y)_{y=Y} + (u_y)_{y=0}}{2} \tag{46}$$

When taking into consideration:

$$(u_y)_{y=0} = 0$$

starting condition, on the basis of Equations (44), (45) and (46) we obtain:

$$K_v \frac{d^2\varepsilon'}{dr^2} = \frac{u'_y}{2Y} \tag{47}$$

By introducting relative variables and the dimensionless expression:

$$A_6 = \frac{R^2 u_e}{2YK_v\bar{\varepsilon}} \tag{48}$$

and considering identity:

$$\varepsilon + \varepsilon' \equiv 1 \tag{49}$$

from Equations (32) and (47) the following formula is obtained:

$$\frac{d^2\varepsilon_r}{dr_r^2} = A_6 \frac{0.85\ (u'_{ry} - 1)}{1 + \sqrt{1 + \frac{3.4\ \bar{\varepsilon}(u'_{ry} - 1)}{A_4}}} \tag{50}$$

Equations (18), (32) and (50) can be regarded the three basic equations which enable the fluid mechanical properties (fluid flow rate between the particles, particle velocity and changes in void fraction) to be described.

The Effect of Layer Viscosity

In the foregoing, the influence of the viscosity of the layer was not taken into consideration.If we were to endeavour to formulate a correct mathematical model, our equations would become complicated to such an extent that their solution would be ab ovo impossible. Accordingly, a very rough approximation, described in the following, was adopted. This approximation is, however, a true model of the systems, as far as the tendencies are concerned. A fluidized layer, a uniform streaming system, will be considered This system streams in the tube as a consequence of the force exerted by the fluid upon the particles. The simplified Navier--Stokes equation (10) can be applied for the description of this process, very much as was done in the description of the streaming of the fluid, the only difference being that in this case it is assumed that the pressure of the layer is zero due to the floating of the particles.The solution of the equation enables the particle velocity in the middle of the layer to be determined in the case of a viscous layer and also if the viscosity is zero. The proposed approximation is that the ratio of the two velocities in the case of a fluidized layer is equal to the velocity ratio thus obtained. Accordingly, the equation - based on Equations (16) and (19), and taking the force acting upon the particles into consideration - will be the following:

$$(\rho - \rho')g\varepsilon(u'_{ry} - u_r) - (\rho - \rho')g\varepsilon = \frac{\mu_{fl}u_e}{R^2}\left(\frac{d^2u_r}{dr_r^2} + \frac{1}{r_r}\frac{du_r}{dr_r}\right) \quad (51)$$

Considering the state when the particles are no longer accelerated. In this case, taking Equation (22) into consideration, it is true that:

$$u'_{ry} - \bar{u}_{ry} = 1 \tag{52}$$

Introducing a new symbol:

$$u_r =. \bar{u}_{ry} + \Delta u \tag{53}$$

From Equations (51), (52) and (53) we have:

$$\Delta u(\rho - \rho')g\varepsilon = \frac{\mu_{fl}u_e}{R^2}\left[\frac{1}{r_r}\frac{d(r_r \frac{du_r}{dr_r})}{dr_r}\right] \tag{54}$$

Boundary conditions:

$$\left(\frac{du_r}{dr_r}\right)_{r_r=0} = 0, \quad u_r(1) = 0 \tag{55}$$

By neglecting the changes in particle velocity difference along the radius:

$$\frac{d\Delta u}{dr_r} \approx 0 \tag{56}$$

By the introduction of the dimensionless quantity:

$$A_5 = \frac{\mu_{fl}u_e}{(\rho - \rho')\bar{\varepsilon}gR^2} \tag{57}$$

the solution of Equation (54) is:

$$u_r = \frac{u_{ry}}{1 + A_5} \tag{58}$$

In accordance with the foregoing, u_r is written in place of u_{ry}.

= u_y/u_e in Equation (18), $1.7/(1 + A_5)$ is written in place of 1.7 in Equation (32), and $A_6/(1 + A_5)$ is written in place of A_6 in Equation (50).

Changes in the Liquid Flow Rate between the Particles and in the Particle Velocity along the Radius

In the case of fluidization with a liquid it can be shown by substitution of numerical values that:

$$u_e \ll g\tau \tag{59}$$

By taking this into consideration, from Equation (26) we obtain:

$$y = (u'_{ry} - 1)u_e\tau \tag{60}$$

Accordingly, the mean relative particle velocity - taking the effect of viscosity into consideration - is the following:

$$u_r = \frac{(u'_{ry} - 1)}{1 + A_5} \tag{61}$$

Accordingly, from Equation (50) we obtain:

$$\frac{d^2\varepsilon_r}{dr_r^2} = A_6 \frac{u'_{ry} - 1}{2} \tag{62}$$

In the case of fluidization with a liquid, the value of A_6 is low and consequently it follows from Equation (62):

$$\frac{d^2\varepsilon_r}{dr_r^2} \approx 0 \tag{63}$$

On account of the cylindrical symmetry:

$$\left(\frac{d\varepsilon_r}{dr_r}\right)_{r_r=0} = 0 \tag{64}$$

and consequently it follows from Equations (63) and (64), since $(\varepsilon_r)r_r = 1$ that:

$$\varepsilon_r \equiv 1 \tag{65}$$

On the basis of Equation (11), if y = Y:

$$A_1 = \frac{\Delta p}{[(\rho - \rho')\bar{\varepsilon} + \rho'\bar{\varepsilon}']gY} \tag{66}$$

From basic Equation (18), considering Equations (61) and (65) we obtain:

$$A_1[\bar{\varepsilon}(\rho-\rho') + \bar{\varepsilon}' - \rho']g = \mu' \frac{1}{r}\frac{d}{dr_r}\left(r_r\frac{du'_{ry}}{dr_r}\right) + g(\rho-\rho')\bar{\varepsilon} + \bar{\varepsilon}'\rho'g \tag{67}$$

By introducing the dimensionless quantity:

$$A_2 = \frac{\mu' u_e}{R^2[(\rho - \rho')\bar{\varepsilon} + \rho'\bar{\varepsilon}']g} \tag{68}$$

and rearranging Equation (67) we obtain:

$$(A_1 - 1) = A_2 \frac{1}{r_r}\frac{d}{dr_r}\left(r_r\frac{du'_{ry}}{dr_r}\right) \tag{69}$$

Boundary conditions:

$$\left(\frac{du'_{ry}}{dr_r}\right)_{r_r=0} = 0; \quad u'_{ry}(1) = 0 \tag{70}$$

The solution of Equation (69) is the following:

$$u'_{ry} = \frac{1 - A_1}{4\ A_2} (1 - r_r^2) \tag{71}$$

With the use of Equation (61), the following formula is obtained from Equation (71):

$$u_r = \frac{1}{1 + A_5} \left[\frac{1 - A_1}{4\ A_2} (1 - r_r^2) - 1 \right] \tag{72}$$

The volume of the particles moving upwards or downwards during unit time is identical; accordingly:

$$\int_0^1 2\pi r_r \bar{\varepsilon} u_r dr_r = 0 \tag{73}$$

From Equations (72) and (73):

$$\frac{1 - A_1}{4\ A_2} = 2 \tag{74}$$

The liquid flow rate between the particles along the radius - on the basis of Equations (71) and (74) - is the following:

$$u'_{ry} = 2\ (1 - r_r^2) \tag{75}$$

The change in particle velocity - on the basis of Equations (72) and (74) - is:

$$u_r = \frac{2\ (1 - r_r^2) - 1}{1 + A_5} \tag{76}$$

The maximum relative particle velocity:

$$u_{rM} = \frac{1}{1 + A_5} \tag{77}$$

CALCULATION OF THE VOID FRACTION IN FLUIDIZATION WITH A LIQUID

On the basis of geometric considerations the following identities are valid:

$$\frac{\pi d^3}{6\ d_I^3} \equiv \varepsilon \tag{78}$$

$$\frac{\pi d^2}{4\ d_I^2} \equiv E \tag{79}$$

It follows from Equations (78) and (79):

$$\frac{8}{6\sqrt{\pi}}\ E^{3/2} \equiv \varepsilon \tag{80}$$

By applying the continuity rule to the streaming of the liquid:

$$\int_o^R 2\pi u_y' E' dr = \pi R^2 U' \tag{81}$$

Considering Equations (65) and (80):

$$E' u_e = U' \tag{82}$$

using the identity:

$$E + E' \equiv 1 \tag{83}$$

we obtain from Equation (82):

$$U' = u_e(1 - E) \tag{84}$$

The particle volume fraction can be obtained from Equation (84) with the application of Identity (80):

$$\varepsilon = \frac{8}{6\sqrt{\pi}}\left(1 - \frac{U'}{u_e}\right)^{3/2} = 0.75\ (1 - U'_r)^{3/2} \tag{85}$$

The formula is valid even at the minimum fluidization velocity:

$$\varepsilon_m = 0.75\ (1 - U'_{rm})^{3/2} \tag{86}$$

The void fraction can be calculated from Equations (85) and (86), considering also Equations (49):

$$1 - \varepsilon' = \frac{1 - \varepsilon'_m}{(1 - U'_{rm})^{3/2}}\ (1 - U'_r)^{3/2} \tag{87}$$

The results of validity tests carried out in connection with Equations (85) and (87), derived for the calculation of the expansion of fluidized layers, will be reported on in the next paper of this series

USED SYMBOLS

- a constants
- A composite, dimensionless characteristics
- d diameter of the particles (m)
- E cross section fraction of the particles (m^2/m^2)
- E' free cross section fraction (m^2/m^2)
- F cross section of the layer (m^2)
- g graviational acceleration (m/sec^2)
- K_v vibration coefficient (m^2/sec)
- p pressure ($kg/m\ sec^2$)
- P force acting upon a unit volume ($kg/m^2/sec^2$)
- P_y force acting upon a unit volume in the direction y (kg/m^2sec^2)

P_φ force acting upon a unit volume in the direction φ (kg/m^2sec^2)

P_R force acting upon a unit volume in the direction R (kg/m^2sec^2)

r radius co-ordinate (m)

R radius of the layer (m)

u_e fall velocity of the particles (m/sec)

$\bar{u}_y$ mean velocity of the particles along the height, if the viscosity of the layer is zero (m/sec)

$\bar{u}$ mean particle velocity (m/sec)

u_y particle velocity along the height (m/sec)

u_v vibration rate (m/sec)

u_R particle velocity along the radius (m/sec)

u' flow rate of the medium (m/sec)

$\bar{u}'_y$ mean flow rate of the fluid along the height (m/sec)

u'_R flow rate of the fluid along the radius (m/sec)

u'_φ flow rate of the fluid along an angle (m/sec)

u'_y flow rate of the fluid along the height (m/sec)

U' feed rate of the fluid (m^3/m^2sec)

U'_m minimum feed rate of fluid (m^3/m^2sec)

v volume of the particles (m^3)

y height co-ordinate (m)

Y layer height (m)

Y_m minimum layer height (m)

Y_v vibrational lenght (m)

Δp pressure difference ($kg/msec^2$)

ε particle volume fraction (m^3/m^3)

$\bar{\varepsilon}$ mean particle volume fraction (m^3/m^3)

ε_m minimum particle volume fraction (m^3/m^3)

ε_r relative particle volume fraction (dimensionless)

ε' void fraction (m^3/m^3)

$\bar{\varepsilon}'$ mean void fraction (m^3/m^3)

ε'_m minimum void fraction (m^3/m^3)

ε'_r relative void fraction (dimensionless)

μ' viscosity of the fluid (kg/sec m)

μ_{fl} viscosity of the fluidized layer (kg/sec m)

ρ density of the solid phase (kg/m^3)

ρ' density of the fluid (kg/m^3)

τ time (sec)

φ angular co-ordinate (degree)

Markings

A straight line drawn over the symbol: $\bar{}$: mean value.

A letter "r" on the lower right-hand side of the symbol: relative quantity.

No marking on the upper right-hand side of the symbol: solid phase.

A comma on the upper right-hand side of the symbol: ': fluid: liquid or gas.

Relative variables

$$u_r = \frac{u}{u_e} \qquad u_{ry} = \frac{u_y}{u_e}$$

$$u'_r = \frac{u'}{u_e} \qquad u'_{ry} = \frac{u'_y}{u_e}$$

$$U'_r = \frac{U'}{u_e} \qquad U'_{rm} = \frac{U'_m}{u_e}$$

$$r_r = \frac{r}{R} \qquad \bar{u}_{ry} = \frac{\bar{u}_y}{u_e}$$

$$\varepsilon_r = \frac{\varepsilon}{\bar{\varepsilon}} \qquad u_{rM} = \frac{u_M}{u_e}$$

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РЕЗЮМЕ

Исходя из физической модели, авторы излагают вывод основных уравнений для движения зерен и измерения плотности зерен вдоль радиуса, пригодных для описания гидродинамических условий псевдоожиженных систем.

Выведенные уравнения применены авторами для систем псевдоожиженных с помощью жидкости, представлены ими соотношения для вычисления скорости жидкости между зернами, скорости зерен вдоль радиуса, а также и доли свободного объема.