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APPLICATION OF ANALOG COMPUTERS MEASUREMENT AND EVALUATION OF RESIDENCE TIME DISTRIBUTION

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The authors describe a measurement and calculation method which enables direct contact to be made between any optional operational unit and an analog computer. The calculation and evaluation of residence time distribution in the case of absorbers is discussed as an example.

In connection with the above, the expressions which became usual in connection with residence time distribution are modified and transformed in such a way as to obtain quantities everywhere which can be directly fed into an analog computer.

An advantageous property of the method is its high speed, because calculations can be carried out simultaneously with the experiments. In addition, any other variable which is convertible to an electric tension can be directly fed into an analog computer. For example, a chromatograph can be directly connected to an analog machine. Accordingly, concerning its technical utilization the method is also universal.

The residence time distribution of the flowing phases is of paramount importance in the design and operation of chemical industrial operational units.

The above-mentioned fact has been discussed in a paper of basic importance by DANCKWERTS [1]. He defined the most important concepts connected with residence time distribution. It is sufficient to quote his important statement which can be summarized in the following manner: if the residence times of the individual elements of the flowing phases are different, i.e. they vary according to some sort of distribution, the efficiency of the operational unit (apparatus) is always lower compared to that observed in the case of the identical residence time of the elements. A number of concepts - defined on the basis of DANCKWERTS' paper - are at present widely applied [2 - 7]. These concepts are assumed to be known and they are summarized in Table 1. The only remark to be made in connection with this is that two concepts for time are usually used: t is the time actually measured, whereas θ is dimensionless time referred to the $\bar{t}$ mean or apparent residence time. The connection between these two quantities is expressed by the following equation:

$$\theta = \frac{t}{\bar{t}} = \frac{tv}{V} \qquad (1)$$

Table 1.

Denomination	Experimental curves		Theoretical curves
density function	$C(\theta) = \frac{dF}{d\theta}$	$\longrightarrow$	$E(\theta) = \frac{dI(\theta)}{d(\theta)}$
distribution function	$F(\theta) = \int_0^{\infty} C(\theta)\, d\theta$	$\longrightarrow$	$1 - I(\theta) = \int_0^{\theta} E(\theta)\, d\theta$
intensity function	-		$\Lambda(\theta) = \frac{E(\theta)}{I(\theta)} = \frac{d}{d\theta} \ln I(\theta)$

As to the method of measurement it should be noted that two techniques are used generally: 1. The tracer substance is added to the input stream for only a short period of time in a pulse-like manner, whereupon its addition is disconnected. The c(t) function of the tracer substance, being the response to the above input pulse, is measured or recorded in the output stream (Fig.1.).

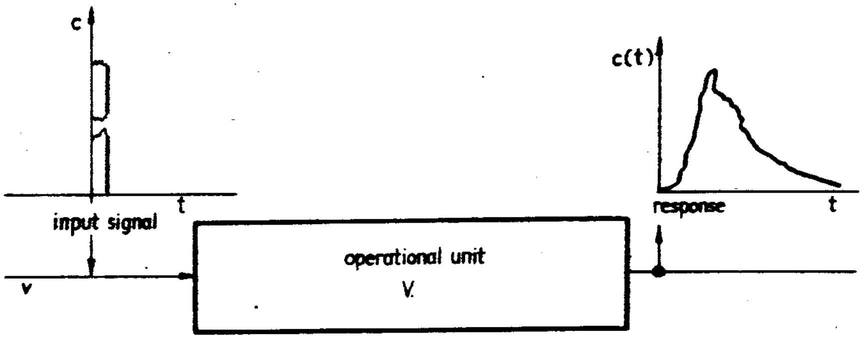

Fig.1. Schematical representation on the study of residence time distribution in the case of pulse-like signal.

2. The tracer substance is added to the input stream at a well-defined (c_o) concentration at a time t = 0 and this (c_c) concentration is maintained in the following (Fig. 2.). This type of signal is

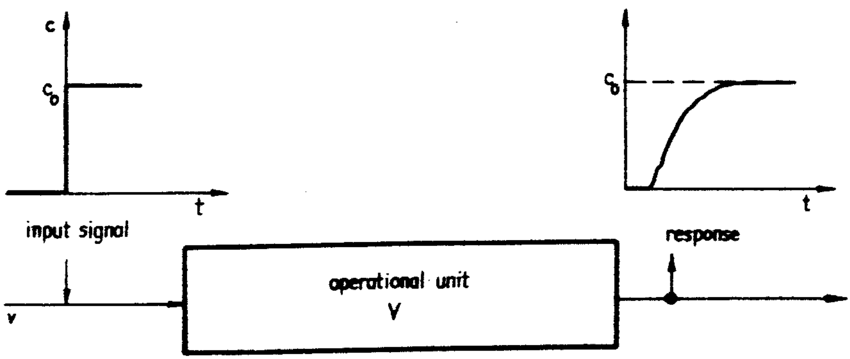

Fig.2. Schematical representation of the study of residence time distribution in the case of jump-like signal.

often called a "jump-like input signal". In such a case, the $c(t)$ concentration of the tracer substance is measured or recorded against time in the exit stream from a time $t = 0$. In this case the limiting value of the response function $c(t)$ is $c(t) \to (c_o)$ if $t \to \infty$. The ratio of $c(t)$ and (c_o) is the distribution function of the residence time:

$$\frac{c(t)}{c_o} = F(t) \tag{2}$$

In practice, instead of the $c(t)$ function it is advantageous to measure its value multiplied by the volumetric flow rate (v) and divided by the amount of tracer substance brought into the system (Q). This quantity is called function $C(t)$:

$$\frac{c(t)v}{Q} = C(t) \tag{3}$$

The function $C(t)$, as determined experimentally, is identical to the theoretical $E(t)$ residence time distribution density function, if the tracer substance brought into the system truly represents the elements of the streaming phase at the exit side.

According to the well-known laws of mathematical statistics, the connection between the functions described in the foregoing is the following:

$$C(t) \to E(t) = \frac{d\,F}{d\,t}$$

and

$$C(\theta) \to E(\theta) = \frac{d\,F}{d\,\theta} \tag{4}$$

It should be noted that these concepts and measuring methods can not only be applied in connection with continuous operational units of chemical engineering, but also in connection with any other type of engineering activity, where different media flow through any apparatus of a given size and shape, such as, for example, sewage purification and settling tanks.

DESCRIPTION OF THE MEASURING METHOD

The determination of the functions and integrals shown in Table 1. is generally a cumbersome and time-consuming task. Accordingly a measuring technique and a programme was elaborated, the essence is that the experimental data measured in the operational unit are, after suitable and proportional transformation, fed into an analog computer. The latter carries out the calculations simultaneously with the measurement.

The experiments were concerned with adsorption processes occurring in packed columns. However, it should be stressed that the method is of a general nature and can be applied to any operational unit. The calculations were carried out with the momentum method [8, 9, 10].

DESCRIPTION OF THE EXPERIMENTS

The liquid phase applied in the packed column was water and the tracer substance was aqueous NaCl solution. The conductance of the effluent liquid was measured. In the concentration range used in these experiments, the connection between conductance and salt concentration of the liquid is linear. Experiments were carried out with pulse-like and jump-like changes in the amount of the tracer substance.

The connection between the operational unit and the analog computer was established according to Fig. 3. A detector is placed into the liquid leaving the operational unit and the response of the apparatus to the interfering signal is recorded by a potentiometric strip-chart recorder. The sensing device was a conductivity meter equipped with platinum electrodes. A follower potentiometer of 10 kiloohms resistance - this value being matched to the analog computer - was coupled to the shaft actuating the sliding contact of the measuring bridge built into the recorder. A potential

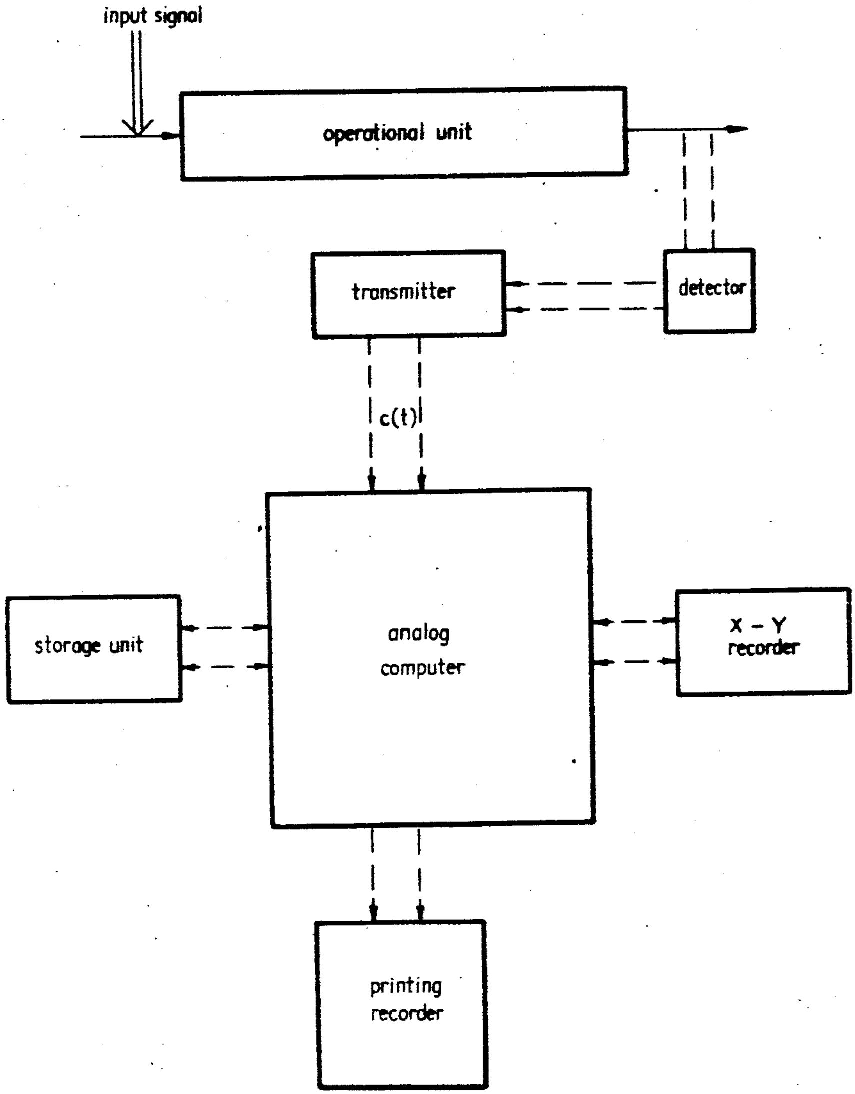

Fig.3. Schematic representation of the connection of the operational unit to the analog computer.

of 10 volts - again matched to the computer - was fed to the end points of the potentiometer. The sliding contact of this potentiometer followed that of the recording potentiometer and,consequently a voltage proportional to that measured by the recorder, i.e. to the conductance of the liquid was obtained between one end and the sliding contact of the follower potentiometer. This voltage can be regarded as equal to the function c(t) and this was fed into the analog computer which carried out the necessary calculations.

It should be noted that the X-Y recorder, which is an accessory of the computer type MEDA 81 T, can also be utilized as a curve reader. Accordingly, the records representing the results can be used in the calculations.

CALCULATIONS

1. Pulse-like Input Signal. The first momentum gives the mean value of the distribution, which in case of a c(t) distribution function is identical to the mean residence time ($\bar{t}$):

$$\bar{t} = \frac{\int_0^\infty t\, c(t)\, dt}{\int_0^\infty c(t)\, dt} = \int_0^\infty t\, C(t)\, dt \qquad (5)$$

The variance σ^2 can be calculated from the experimental data according to the following formula:

$$\sigma^2 = \frac{\int_0^\infty t^2\, c(t)\, dt}{\int_0^\infty c(t)\, dt} \qquad (6)$$

The central variance σ_c^2 is, from the above:

$$\sigma_c^2 = \sigma^2 - \bar{t}^2 \qquad (7)$$

2. Jump-like Input Signal. In this case, the calculation of the momenta is carried out as follows:

The definition of the mean residence time is, according to Eq. (2),

$$\bar{t} = \frac{\int_0^\infty t \; d\; c(t)}{c_o} = \int_0^\infty t \; d \; F(t) \tag{8}$$

The mean residence time cannot be determined with an analog computer. The independent variable is always time and, consequently integration can be carried out only according to time. The expressions referring to a jump-like input signal must be transformed in such a way as to contain integration according to time.

When taking Equation (4) into consideration, Equation (8) can be resolved into two parts:

$$\int_0^\infty t \; d \; F(t) = \int_0^\infty t \; C(t) \; dt = \left[t \; F(t) \right]_0^\infty - \int_0^\infty F(t) \; dt$$

In accordance with practical considerations, the integration is not carried out until infinite time, but a sufficiently large t_m value is chosen at which the value of $F(t_m)$ is practically identical to $F(t_\infty) = 1$. Consequently the value of the integral is t_m and

$$\bar{t} = t_m - \frac{\int_0^{t_m} c(t) \; dt}{c_o} \tag{9}$$

This expression enables the calculation of the mean residence time from experimental data by means of an analog computer.

The definition for the calculation of the variance is the following:

$$\sigma^2 = \int_0^\infty t^2 \; d \; F(t)$$

which, transformed in a way analogous to that described in the forregoing, can be brought to a form allowing the calculations:

$$\sigma^2 = t_m^2 - 2 \frac{\int_0^{t_m} t\, c(t)\, dt}{c_o} \quad (10)$$

Similarly, for the central variance we have:

$$\sigma_c^2 = t_m^2 - 2 \frac{\int_0^{t_m} t\, c(t)\, dt}{c_o} - \bar{t}^2 \quad (11)$$

The following experimental data have to be known to calculate the integrals described in Equations (5), (6) and (7), as well as (9), (10) and (11):

Pulse-like input signal:	Jump-like input signal:	
$\int_0^{\infty} c(t)\, dt = I_1'$	$c_o = Y_1'$	(12)
$\int_0^{\infty} t\, c(t)\, dt = I_2'$	$\int_0^{t_m} c(t)\, dt = Y_2'$	(13)
$\int_0^{\infty} t^2\, c(t)\, dt = I_3'$	$\int_0^{t_m} t\, c(t)\, dt = Y_3'$	(14)

Direct calculation of these quantities was also possible if time t was at our disposal in the form of a potential. This method permits a voltage proportional to time to be produced. Indicating the voltage proportional to time by U_t:

$$U_t = K_1 \cdot t \quad (15)$$

and this is the solution of the following differential equation:

$$\frac{d\, U_t}{dt} = K_1 \quad \text{and} \quad U_t(o) = 0 \quad (16)$$

Accordingly, the function $U_t(t)$ can be produced with the programme shown in Fig.4.

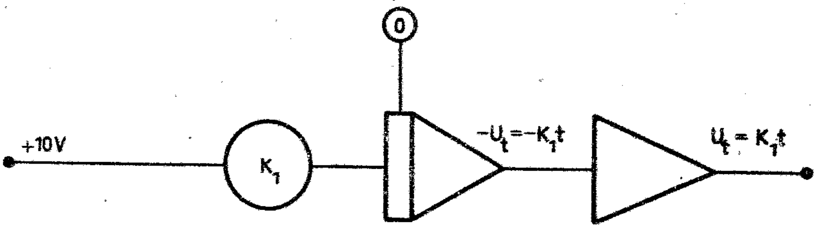

Fig.4. Programme for the production of a voltage proportional to time.

After this, the necessary integrals can be calculated. The computer programme is shown in Fig.5. Constants k_1, k_2, k_3 and k_4 are scale factors. Their values are to be set during the measurement in such a way that the potential produced at the computing units should not be higher than that compatible with the computer. Fig.5. indicates on which units the values of the integrals are read. These do not directly give the values (12), (13) and (14), but rather their products with a constant:

$$I_1' = \frac{I_1}{k_2}$$

$$I_2' = \frac{1}{k_1\ k_4}\ I_2$$

$$I_3' = \frac{1}{k_1^2\ k_3}\ I_3$$

The values of I_1, I_2 and I_3 are read from the computer at the end of the experiment and the momenta can be calculated with the following simple equations:

In the case of a pulse-like input signal:

$$\bar{t} = \frac{k_2}{k_1\ k_4}\ \frac{I_2}{I_1} \qquad (17)$$

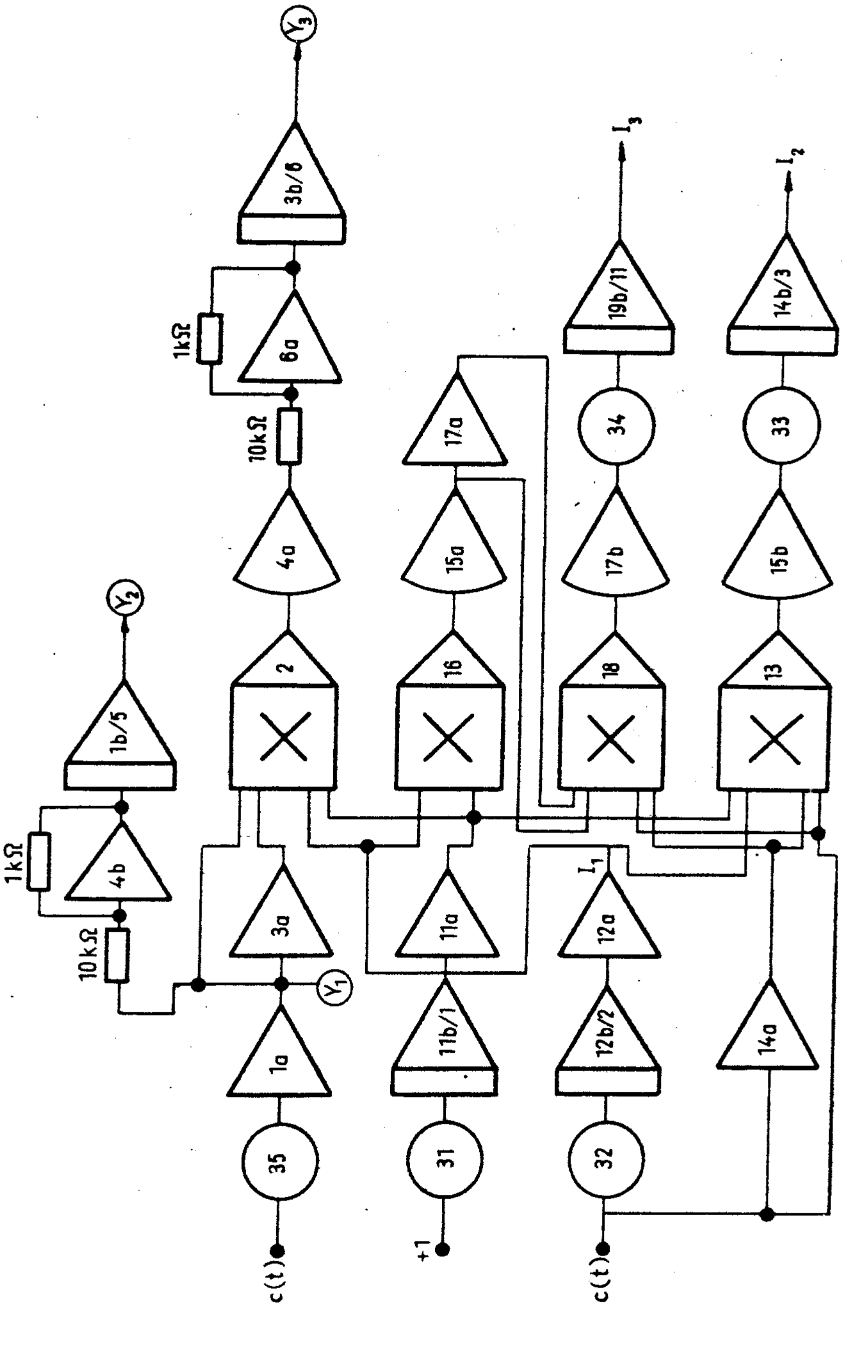

Fig.5. Programme for evaluation of the experiments

$$\sigma^2 = \frac{k_2}{k_1^2 k_3} \frac{I_3}{I_1} \tag{18}$$

In the case of a jump-like input signal:

$$\bar{t} = t_m - \frac{Y_2}{Y_1} \tag{19}$$

$$\sigma^2 = t_m^2 - \frac{Y_3}{k_4 k_1 Y_1} \tag{20}$$

The response curve obtained with a pulse-like input signal together with the calculated values, based on experiments carried out with a packed column, are shown in Fig.6.

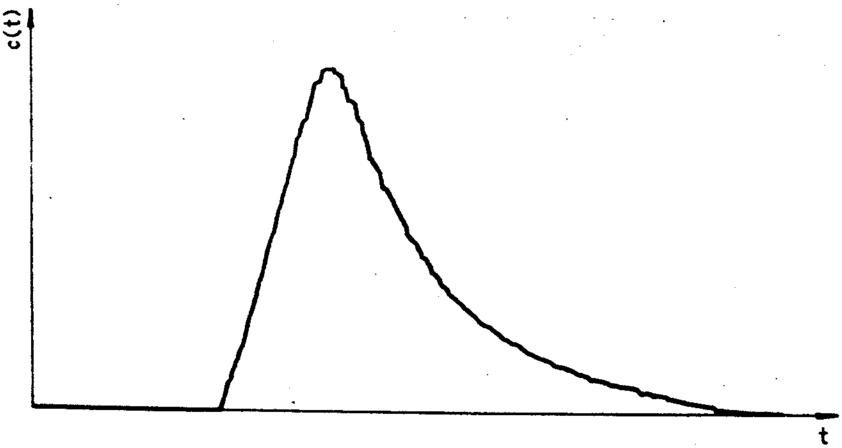

Fig.6. c(t) curve obtained from the analog computer and result of the calculation in the case of pulse-like disturbance

I_1 = 0.108 $\bar{t}$ = 43.9 sec

I_2 = 0.237 σ^2 = 2088 sec^2

I_3 = 0.364 σ_c^2 = 163.4 sec^2

The response curve obtained with a jump-like input signal, as observed in experiments carried out with a liquid phase streaming in a rotating film-reactor, are shown in Fig.7.

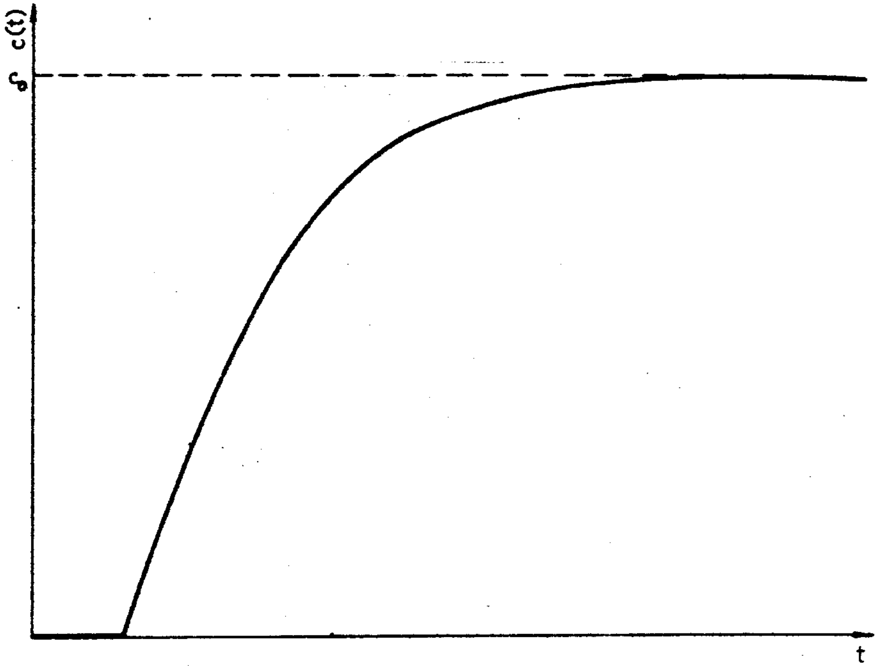

Fig.7. c(t) curve obtained from the computer and result of the calculation in the case of jump-like disturbance

$Y_1 = 0.067$ $\bar{t} = 29$

$Y_2 = 0.489$ $\sigma^2 = 1164$

$Y_3 = 0.296$ $\sigma_c^2 = 435$

Calculations can be carried out by the computer simultaneously with the measurements. The integrals I_1, I_2 and I_3, as functions of time, are present in the computer and their values can be read, printed out or recorded continuously.

According to the discussed method, the analog computer can be applied to construct the functions themselves. For example, in the case of a pulse-like input signal, the value of the integral I_t is proportional to the functions F(t) and I(t) (cf. Table 1.), since the function F(t) can be expressed in the following form:

$$F(t) = \frac{\int_0^t c(t)\,dt}{\int_0^\infty c(t)\,dt} = \frac{I_1\ (t)}{I_1\ (\infty)} \tag{21}$$

Consequently, the function F(t) can be directly obtained from the computer or recorded in the case of a pulse-like input signal.

LIST OF SIGNS

C(t) density function of experimentally determinable residence time distribution (sec^{-1})

c(t) response function furnished by the operational unit upon disturbance ($moles/m^3$)

c_o the value of the jump in the case of jump-like disturbance ($moles/m^3$)

E(t) density function of residence time distribution (sec^{-1})

F(t) distribution function of residence time (dimensionless)

I(t) age distribution function (dimensionless)

I_1 value read on analog computer (cf. Fig.5.)

I_2 value read on analog computer (cf. Fig.5.)

I_3 value read on analog computer (cf. Fig.5.)

I_1' cf. Equation (12)

I_2' cf. Equation (13)

I_3' cf. Equation (14)

k_1, k_2, k_3, k_4 scale factors (cf. Fig.5.)

t time (sec)

$\bar{t}$ mean residence time (sec)

V volume of operational unit (m^3)

v volumetric flow rate of phase (m^3/sec)

Q amount of pilot substance injected (moles)

Y_1 cf. equation (12)

Y_2 cf. equation (13)

Y_3 cf. equation (14)

$\Lambda(t)$ intensity function (sec^{-1})

σ^2 second momentum (sec^2) or variance

σ_c^2 second central momentum (sec^2)

REFERENCES

1. DANCKWERTS, P.V., Chem. Eng. Sci. 2, 1 (1953).
2. LEVENSPIEL, O., BISCHOFF, K.B., Adv. Chem. Eng. 4, 95 (1963).
3. KAFAROV, V.V., Metodi kibernetiki v himiya i himitsheskoy technologii, Izd. "Himiya", Moscow (1968).
4. BISCHOFF, K.B., McCRACHEN, A.E., Ind. Eng. Chem. 58, 18 (1966).
5. DANCKWERTS, P.V., Ind. Chemist. 3, 102 (1954).
6. NAOR, P., SINNAR, R., Ind. Eng. Chem. Fund. 2, 278 (1963).
7. HIMMELBLAU, D.M., BISCHOFF, K.B., Process Analysis and Simulation, John Wiley and Sons Inc., New York 1968.
8. van der LAAN, E.Th., Chem. Eng. Sci. 7, 1 (1957).

9. ÁRVA, P., Töltelékes oszlopban történő kétfázisu áramlás matematikai modellezése. Kandidátusi értekezés. (The Mathematical Modelling of Two Phase Flow in Packed Columns. Candidate Thesis.) Moscow, 1970.

10. ARIS, R., Introduction to the Analysis of Chemical Reactors. Prentice-Hall. Englewood Cliffs, N.J. 1965. p. 205.

РЕЗЮМЕ

Автсрами описан метод измерения и вычисления, пригодный для создания непосредвтвенной связи между любым элементом процесса и аналоговой ВМ. В качестве примера показано вычисление и оценка распределения времени пребывания жидкости в случае абсорберов.

В связи с этим изменены выражения, образовавшиеся для распределения времени пребывания, и они превращены таким способом, чтобы везде сфигурировались величины,непосредственно вводимые в ВМ.

Праимуществом метода является его высокая скорость, так как вычисления выполняются одновременно с опытами. Кроме этого, данный метод пригоден к непосредственному введению в АВМ любой другой переменной, превратимой в электрическое напряжение. Так, напр., и хроматограф можно непосредственно подключить к аналоговой АВМ. По этому метод является всеобщим и с точки зрения технологического выполнения.