

APPLICATION OF FOURIER TRANSFORMATION FOR WASTE MINIMIZATION IN BATCH PLANTS. 2. PROCESS-UNITS ASSIGNMENT

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The problem for determining the minimum environmental impact for compatible products manufacturing in multipurpose batch plants is considered in this study. It is based on the use of the Fourier transformation for mathematical descriptions of the waste emissions from routine sources appearing into the horizon cyclically - an approach which has been proposed in its first part [4]. The problem takes into accounts both the used materials compositions and the constructed production routes. The formulated sets of constraints follow for feasibility and compatibility of the chosen production routes and justify the accomplishment of the production demands into the determined horizon. Global or Local Environmental Impact Assessments are used as the objective function.

An example concerning simultaneous manufacturing in a dairy of two types of curds is used to illustrate the considered problem. The aim is to determine the milkfat content in the skimmed milk used as a raw material for both products, and plant units assignment for the respective processing tasks, at which the *BOD* generated from the process is minimal for accomplishment of some production requirements in a given horizon. Both the *BOD* generated due to the amount and composition of the processed milk and the one due to inherent losses are taken into account in the formulated problem.

Keywords: Waste minimization, Fourier transformation, Multipurpose batch plants, *BOD*, Dairy processing

Introduction

Following contemporary trends to reduce the environmental impact by developing systematic methodologies for waste minimization in sources, Pistikopoulos at al. have created the Minimum Environmental Impact Methodology for continuous and batch plants [1-3]. They have embedded the Life Cycle Analysis principles within a design and optimization framework and introduced appropriate quantitative environmental impact assessments. The latter are defined on the basis of the introduced environmental impact indices, such as CTAM, CTWM, SMD, GWI, POI, SODI, etc., and determine the environmental impact of the whole

system or of a particular pollutant over the time horizon.

Later, in the first part of the current study [4], an approach describing the wasting from the batch routine sources has been proposed. It is based on the transformation of the periodic discontinuous function of the waste mass rate of pollutants w in the Fourier series into the time horizon and allows for the relevant pollutant to be followed within the horizon. The obtained continuous time function presents a mathematical model of the waste producing from the routine source, which appears cyclically into the horizon during batch product manufacturing. The model involves the general characteristics of the batch product, such as batch size, cycle time, and processing times as well as accounts for the raw materials composition. It can

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be propagated readily not only to all the routine sources of pollutant - w at a current manufacture, but also to those, appearing at the compatible processing of a group and campaign of batch products in the definite horizon. Its integration over the time allows for the determination of the produced waste - w from the relevant routine sources. This permits the use of the environmental impact assessments introduced in [1-3]. As an illustration, the proposed approach has been applied for production recipes analysis of examples from dairy industry (curds processing). The aim was to determine the raw material composition at which the environmental impact, assessed by BOD, for manufacturing of 1-kilogram target product was minimal.

In order to reduce the waste from the multipurpose batch plant, at accomplishing the production demands for a group of compatible products, both the raw materials compositions and the process/units assignments must be taken into account. They affect not only the production characteristics, such as batch size and the number of processed batches, but also the environmental impact through the amount of the waste produced. Usually, production and environmental requirements are in conflict. The objective of this study will be to find the best tradeoff between them. It will be based on the application of the approach for mathematical modeling of the wasting from appearing cyclically routine sources proposed in the first part. The aim will be reached through simultaneously determining the used materials compositions and the process/units assignments at which the production demands will be fulfilled into the given horizon with minimum environmental impact.

The paper is structured as follows: A description of the problem is presented in next part. A mathematical formulation of the waste minimization problem is laid out in part 3. An application of the proposed approach on an example from the dairy industry – simultaneous processing of two types of curds into a given plant is presented in part 4, while the concluding remarks are in part 5.

Problem description

Let us consider a multipurpose batch plant comprising P units $p \in P$ of different types. Each unit has a volume V_p , $p \in P$ and could be connected with others in the plant. The plant provides an opportunity for a compatible manufacturing of I different products, $i \in I$ within a given time horizon H . Q_i [kg] is the production demand for the respective products i .

Each product i comprises L_i , processing tasks, $l \in L_i$. A set of N_i key components $n \in N_i$ is used for product manufacturing. The key components are introduced into the process by the raw and/or other supporting materials. Only one material source for any particular key component n is allowed. The composition of the key components could be changed within the technologically defined boundaries $x_{min}(i)_n$, $x_{max}(i)_n$. Physical, chemical or biochemical transformation takes place in each task until the target product is obtained. The tasks processing times T_{il} are supposed to be constant. The cycle time depends on the chosen operational mode. For the overlapping case it is $TC_i = \max\{T_{il}\}$, while for the non-overlapping - $TC_i = \sum_l T_{il}$.

Each processing task could be performed in one or more appropriate plant units. A variety of process/unit assignments exists, which results in multiple routes for product manufacturing. Binary matrices $ID(i)_{lp}$ are introduced to identify the appropriate plant units for products i as follows: $ID(i)_{lp} = 1$, for units p suitable to process task l , and $ID(i)_{lp} = 0$, for the others.

The volumes of the involved plant units and the size factors for the respective tasks determine the batch size of a structured production route. Usually, the size factors are taken to be constant. But in fact they depend on the used materials composition. The effect of the key components composition on batch size will be taken into account in the problem.

Manufacturing the products is accompanied by producing different waste types. The standard limit values μa_w , μw_w , μs_w , for air, water and soil are

given for the processed pollutants w , $w \in W$. In principle, each task $l \in L_i$ of the products i could be a potential routine source of effluent with pollutants w . The amount of the wastes produced over the time horizon also depends on both the key components composition and the respective batch sizes.

The above description brings the waste minimization problem to determining the composition of raw materials, solvents and other components used in the process and the respective production routes for each product i so that the demands Q could be accomplished into the horizon H at minimum environmental impact.

Mathematical formulation of the optimization problem

Variables and constraints

Variables

A set of continuous variables $x(i)_n$ is introduced for each product i to account for the change of composition of the key components n in the relevant material sources.

A set of binary variables $\zeta(i)_p$ is used to structure the different production routes for each product i , as follows: $\zeta(i)_p = 1$ if unit p is used for product i manufacturing, and $\zeta(i)_p = 0$, otherwise.

Constraints

Structural constraints. The objective of this set of constraints is to structure feasible and compatible production routes for products i .

Manufacturing each product i is feasible, if at least one of the plant units, suitable for tasks l , is assigned in the structured production route:

$$\prod_{l=1}^{L_i} \left(\sum_{p=1}^P ID(i)_{lp} \zeta(i)_p \right) \geq 1 \quad \forall i, i \in I. \quad (1)$$

The structured production routes for all I products are compatible if there are not any common shared units.

$$\sum_{i=1}^I \zeta(i)_p \leq 1 \quad \forall p, p \in P. \quad (2)$$

Production constraints. This group of constraints is introduced to account for the demands Q_i to be accomplished within the time horizon H .

Taking into account that the composition of the key components in the used materials affects both, batch size and environmental impact, admissible vectors $X(i, n)$ of the independent variable values $x(i)_n$ should be formed for each product i ,

$$X(i, n) = \{x(i)_1, \dots, x(i)_n, \dots, x(i)_{N_i}\}. \quad \forall n, n \in N_i \quad \forall i, i \in I, \quad (3)$$

subject to:

$$x \min(i)_n \leq x(i)_n \leq x \max(i)_n, \quad \forall n, n \in N_i \quad \forall i, i \in I. \quad (4)$$

The sizes of batches that are determined by the plant units assigned to the processing task of product i are:

$$B(X(i, n), \zeta(i))_l = \frac{\sum_p V_p \cdot ID(i)_{lp} \cdot \zeta(i)_p}{s(X(i, n))_l}, \quad \forall n, n \in N_i \quad \forall l, l \in L_i, \quad \forall i, i \in I. \quad (5)$$

The size factors $s(X(i, n))_l$ in equations (5) are functions of the key components composition and can be calculated from the mass balances, according to the production recipes:

$$s(X(i, n))_l = \frac{\sum_{j \in l} v(i)_j}{Y(X(i, n))}, \quad j \in l \{l \in L_i\}, \quad \forall l, l \in L_i, \quad \forall i, i \in I, \quad \forall n, n \in N_i \quad (6)$$

where:

$v(i)$ is the volume of the input flow j in the processing task l of product i

$Y(X(i, n))$ is the yield of the target product i presented as a function of the key components compositions.

The batch size for each product i is limited by the processing task with the minimum batch size:

$$B(X(i, n), \zeta(i)) = \min\{B(X(i, n), \zeta(i))_l\}, \quad \forall n, n \in N_i, \quad \forall l, l \in L_i, \quad \forall i, i \in I. \quad (7)$$

The number of batches being carried out in order to manufacture the planned amounts Q_i for the products i is:

$$NB(X(i,n),\zeta(i)) = \left\lceil \frac{Q_i}{B(X(i,n),\zeta(i))} \right\rceil, \quad \forall n, n \in N_i \quad \forall i, i \in I. \quad (8)$$

Finally, the time required to manufacture the demand Q_i for the products i :

$$\Theta(X(i,n),\zeta(i)) = NB(X(i,n),\zeta(i)) \cdot TC_i, \quad \forall n, n \in N_i, \quad \forall i, i \in I, \quad (9)$$

must be within the time horizon H :

$$\Theta(X(i,n),\zeta(i)) \leq H, \quad \forall n, n \in N_i, \quad \forall i, i \in I. \quad (10)$$

$$F(X(i,n),\zeta(i),t)_{wl} = \frac{2B(X(i,n),\zeta(i)) \cdot m(X(i,n))_{wl}}{TC_i} \left[\frac{1}{2} + \sum_k \frac{1}{k\varphi T_{il}} [\cos(k\varphi T_{s_{il}})(1 - \cos(k\varphi T_{il})) + \sin(k\varphi T_{s_{il}})\sin(k\varphi T_{il})] \sin(k\varphi t) + [\sin(k\varphi T_{s_{il}})(\cos(k\varphi T_{il}) - 1) + \cos(k\varphi T_{s_{il}})\sin(k\varphi T_{il})] \cos(k\varphi t) \right]$$

$$0 \leq t \leq TC \quad \forall n, n \in N, \quad \forall w, w \in W, \quad \forall l, l \in L, \quad \forall i, i \in I, \quad (11)$$

where $\frac{2\pi}{TC} = \varphi$.

The mass $m(X(i,n))_{wl}$ of the pollutant w , for products i , can be determined from the pollutants mass balance of the production recipes as it is proposed in [5, 6]:

$$m(X(i,n))_{wl} = \frac{1}{Y(X(i,n))} \left(\sum_{j \in I} MI(i)_j \cdot C(i)_{wj} + R(X(i,n))_{wl} - \sum_{j \in I} MO(i)_j \cdot C(i)_{wj} \right), \quad \forall n, n \in N_i, \quad \forall w, w \in W, \quad \forall l, l \in L_i, \quad \forall i, i \in I \quad (12)$$

where:

$MI(i)$ and $MO(i)$ note the amount of materials input into processing task l of product i by the flows j and output by the flows;

$C(i)$ is the composition of pollutant

$R(X(i,n))$ is the waste produced in the task l .

Mathematical models of the wasting from batch routine source accounting for the composition of the key components and production routes

The mathematical description of the waste w produced from the cycle batch routine source is analogous to that proposed in [4]. The Fourier transformation is used. However, here it must account for the structured particular production routes done through the respective batch sizes, (see equation (5)), and for the key components composition done through both the batch size and the mass $m(X(i,n))_{wl}$ of the pollutant w , generated in the task l for processing of 1 kg. target product:

Objective function

The Local and Global Environmental Impact Assessments introduced in [1-3] are used for the objective function definition. The need for the purpose relevant environmental impact indices such as *STAM*, *WTAM*, *SDM* etc. can be presented by means of the mathematical models (11) introduced above firstly as time dependent functions. Thus, they will account for the used materials composition and process-unit assignment:

$$CTAM(X(i,n),\zeta(i),t)_w = \sum_l \frac{1}{\mu a_w} F(X(i,n),\zeta(i),t)_{wl},$$

$$WTAM(X(i,n),\zeta(i),t)_w = \sum_l \frac{1}{\mu w_w} F(X(i,n),\zeta(i),t)_{wl},$$

$$SMD(X(i,n),\zeta(i),t)_w = \sum_l \frac{1}{\mu c_w} F(X(i,n),\zeta(i),t)_{wl},$$

$$\forall w, w \in W, \quad \forall n, n \in N, \quad \forall l, l \in L, \quad \forall i, i \in I, \quad 0 \leq t \leq TC. \quad (13)$$

The environmental impact indices are obtained after the integration of the equations (13) over the

cycle time duration and multiplying by the number of batches (equations (8)), that can be performed to produce the planned amounts Q_i for products i :

$$\begin{aligned}
 CTAM(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))} &= \\
 &= NB(X(i, n), \zeta(i)) \cdot \sum_l \frac{1}{\mu a_w} \int_0^{TC} F(X(i, n), \zeta(i), t)_{wl} dt \\
 WTAM(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))} &= \\
 &= NB(X(i, n), \zeta(i)) \sum_l \frac{1}{\mu w_w} \int_0^{TC} F1(X(i, n), \zeta(i), t)_{wl} dt \\
 SDM(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))} &= \\
 &= NB(X(i, n), \zeta(i)) \sum_l \frac{1}{\mu s_w} \int_0^{TC} F(X(i, n), \zeta(i), t)_{wl} dt \\
 \forall w, w \in W, \forall n, n \in N, \forall l, l \in L, \forall i, i \in I, \\
 0 \leq t \leq TC. \tag{14}
 \end{aligned}$$

It follows, the relevant Local Environmental Impact Assessments with regard to the particular pollutant - w and respectively Global Environmental Impact Assessment, for all produced pollutants are obtained as:

$$\begin{aligned}
 EI(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))} &= \\
 &= \left[\begin{array}{l} CTAM(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))} \\ CTWM(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))} \\ SDM(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))} \end{array} \right] \\
 \forall w, w \in W, \forall i, i \in I, 0 \leq t \leq TC, \tag{15}
 \end{aligned}$$

$$\begin{aligned}
 GEI(X(i, n), \zeta(i)) \Big|_{\Theta(X(i, n), \zeta(i))} &= \\
 &= \sum_w EI(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))}
 \end{aligned}$$

$$\forall w, w \in W, \forall i, i \in I, 0 \leq t \leq TC. \tag{16}$$

Depending on the particular problem, the Local Environmental Impact Assessments or the Global one could be used as the objective functions in the problems for minimization of the environmental impact of the multipurpose batch plants at the simultaneous production of a group of products.

$$\underset{X(i, n), \zeta(i)}{MIN} EI(X(i, n), \zeta(i))_w \Big|_{\Theta(X(i, n), \zeta(i))}, \tag{17}$$

$$\underset{X(i, n), \zeta(i)}{MIN} GEI(X(i, n), \zeta(i)) \Big|_{\Theta(X(i, n), \zeta(i))}. \tag{18}$$

As a result, the optimal composition of the used raw and other supporting materials and necessary process-units assignment for the production tasks will be obtained.

The non-linear-objective functions (17) or (18), equations (14), (15) (and (16) - at the objective function (18)), the mathematical models of the wasting from routine batch sources (11) and eqs. (12), and as well as constraints (1)-(3) and (5)-(10) represent the problem for environmental impact minimization on the process/unit level. The problem comprises the sets of the two types of independent variables - continuous $x(i)$, forming the vectors - $X(i, n)$ equ. (3), the key components compositions in raw materials, and binary - $\zeta(i)$ structuring the production routes for products i , and constraints in the form of equalities and non-equalities. Its result is the mixed integer non-linear programming (MINLP) problem.

Environmental impact minimization in curds processing

Wastewater is a common factor in the dairy industry that generates a considerable treatment cost. The biological oxygen demand (*BOD*) measures the effluent strength of the wastewater in terms of the amount of dissolved oxygen utilized by microorganisms to oxidize the organic components. The *BOD* load depends not only on the composition and amount of processed whole and/or skimmed milk, but also on the inherent losses due to spilled whey, milk coagulated and glued on the unit's walls, products and by-products lost etc. Since the latter could not be avoided, it is accepted to regulate them to the inherent levels, which are accounted by *BOD* "produced" in the relevant processes.

The example under consideration concerns simultaneous manufacturing in a dairy of two types of curds ($I = 2$), one with a low fat content - 0.3%, called product A; and the other with a high fat content- 1%, - product B. The aim is to determine the milkfat content in the skimmed milk used as a raw material for both products, and plant units assignment for the respective processing tasks, in which the *BOD* generated from the process is minimal for the accomplishment of the posed production requirements in a given horizon.

Curds manufacture is a typical cyclic batch process. For it, processing standard milk skimmed into the boundaries $x_{min}(i) = 0.05\%$ and $x_{max}(i) = 1.4\%$ is used. The key component for both products is the milkfat content $x(i)$, $\forall i \in I$. The composition of standard whole milk is given in the first part of the current study [4]. A detailed report of the curds processing is proposed in [4], too. The description of the processing tasks is presented in *Table 1*. The composition of the target products and values of the respective recovery factors are presented in *Table 2*.

Table 1 Processing tasks description. $CYI(x)^*$ is the yield of curds by-product and x is milkfat content in skimmed milk

Production tasks	Task Duration	Input/Output	Fractions
Task 1 pasteurization	30 min.	In. Skim-milk Out. Pasteurized Skim-milk	1 1
Task 2 acidification	240 min.	In. Skim-milk In. Culture Out. Curds by product Out. Whey	0.88 0.12 $CYI(x)^*$ $1-CYI(x)$
Task 3 draining	30 min.	In. Curds by-product Out. Curds target product Out. Drained Whey	1 0.9 0.1

Table 2 The product compositions and values of the recovery factors

	Composition of the curds target products				Values of the recovery factors		
	Moisture%	FC%	CC%	SC%	RS	RC	RF
A	80	0.3	11.3	20	1.724	0.96	0.075
B	81.58	1.009	12.28	18.42	1.386	0.96	0.231

The Van Slyke equation is used for yield calculation [7]:

$$CY(x(i)) = \frac{[RF \cdot x(i) + RC \cdot MC\%(x(i))] \cdot RS}{SC\%}, \quad \forall i \in I \quad (19)$$

where:

$MC\%(x(i))$ is the casein content in the standardized skimmilk;

RC , RF , RS are the recovery factors for casein, milkfat and other solids different from them;

$SC\%$ is the solids content in the target products.

Since both products involve identical processing tasks, identical relationships determined on the base of the information given in *Table 1* are used for relating them to size factors $s(x(i))_i$:

$$s(x(i)) = \begin{bmatrix} 0.88 \\ CY(x(i)) \\ 1 \\ CY(x(i)) \\ 1.1 \end{bmatrix} \quad \forall i, i \in I. \quad (20)$$

The manufacture of products A and B is carried out simultaneously in a plant comprising the apparatuses listed in *Table 3*. The suitable plant units for the processing tasks of both products are identified by the introduction of the following binary matrices:

Table 3 Plant data

Type	Pasteurizator				Vat reactor			Drainer			
No	1	2	3	4	5	6	7	8	9	10	11
[m ³]	300	250	150	100	300	400	250	80	60	60	100

$$ID(i) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad i = 1, 2. \quad (21)$$

The cycle overlapping operational mode is applied for the products manufacturing, where $TC_i = \max T_i = 4$ [h].

Two types of production demands named Q-I and Q-II are considered for performing in two different horizons H-I and H-II, see *Table 4*:

Table 4 Production demands

Products	Q-I [kg]	Q-II [kg]	H-I [h]	Q-I [kg]	Q-II [kg]	H-II [h]
A	5500	7000	360	5500	7000	400
B	6000	7000		6000	7000	

Each cessing task from the curds production generates the *BOD* due to:

I) The amount and composition of the used skimmed milk. The *BOD* load of 1 kilogram skimmed standard whole milk is determined as follows:

$$BOD_M(x(i)) = 0.89 \cdot x(i) + 1.031 \cdot MP\%(x(i)) + 0.69 \cdot ML\%(x(i)), \quad (22)$$

where:

$MP\%(x(i))$ and $ML\%(x(i))$ are casein and lactose contents presented as a function of the milkfat content; and

II) Associated to the tasks inherent losses [8, 9]:

Task 1 – pasteurization. The pollutant processed is due to the milk coagulated and glued on the pasteurizer's walls. The “generated” *BOD* depends on the amount of the pasteurized milk. The *BOD* load of 1-kilogram pasteurized milk is:

$$BOD_p = 1.5 \cdot 10^{-3} \frac{\text{kg } O_2}{\text{kg pasteurized milk}}. \quad (23)$$

Task 2 – acidification. The pollution results entirely from spilled whey. The inherent leaks are $WL\% = 1.6\%$ from the processed whey mass. The *BOD* load of 1-kilogram acid whey is:

$$BOD_w = 32 \cdot 10^{-3} \frac{\text{kg } O_2}{\text{kg acid whey}}. \quad (24)$$

Task 3 – draining. The polluting is due to:

- i) Draining and discharging of the whey remained in the curds. The *BOD* load of 1-kilogram acid whey is the same as in task 2; and
- ii) Inherent losses of target product gluing on the drainer's wall. They depend on the curds fat content ($FC\%$) - $CL\% = 0.0017 \cdot FC\%$, The *BOD* load of 1-kilogram curds depends on the yield and the *BOD* of used skimmilk:

$$BOD_c(x(i)) = CY(x(i)) \cdot BOD_M(x(i)) \frac{\text{kg } O_2}{\text{kg cheese}}. \quad (25)$$

The mass $m(X(i,n))_{w,l}$ of the pollutant - w , for both products i , is determined from the pollutants mass balance by using the data presented in Table 1 and equations (23)-(25):

$$m(x(i))_{w,l} = \begin{bmatrix} \frac{0.88}{CY(x(i))} & 0 & 0 \\ 0 & \frac{1 - CY(x(i)) \left[1 + \frac{0.1}{0.9} \right] WL\%}{CY(x(i))} & \frac{0.1}{0.9} \\ 0 & 0 & 1 \cdot CL\% \end{bmatrix}, \quad \forall w, \forall l, \forall i \quad (26)$$

The above data and the obtained relations, equations (20) and (26), are completely sufficient to formulate the posed optimization problem, according to equations (1)-(12).

As an optimization criterion the “generated” Global *BOD* in the simultaneous manufacturing of the products A and B is used. It is obtained on the base of the Local *BODs* of the pollutants due to the milk processed, whey spilled and curds lost, from

the respective processing tasks – pasteurization, acidification and draining:

$$BOD(x(i), \zeta(i))_w \Big|_{\Theta(x(i), \zeta(i))} = n(x(i), \zeta(i)) \cdot \sum_{l=1}^3 BOD(x(i))_w \int_0^{TC} F_{wl}(x(i), t) dt, \quad \forall i, i \in I, w = 1, 2, 3. \quad (27)$$

$$\text{where: } BOD(x(i))_w = \begin{bmatrix} BOD_p \\ BOD_w \\ BOD_c(x(i)) \end{bmatrix}.$$

Taking into account equation (27) the objective function - Global Biological Oxygen Demand - *GBOD*, presented as a function of milkfat content - $x(i)$ for products A and B and structured production routes for them by means of vectors $\zeta(i)$ is:

$$\underset{x(i), \zeta(i)}{\text{MIN}} \text{ } GBOD = \sum_{w=1}^3 \sum_{i=1}^2 BOD(x(i), \zeta(i))_w \Big|_{\Theta(x(i), \zeta(i))}. \quad (28)$$

The problem formulated above in the optimization criteria (28) is the environmental impact model for the simultaneous manufacturing of products A and B in the dairy. A – an outcome of its solution the values of the independent variables - milkfat in the skimmilk used for both products and the assigned plant units for the respective tasks are obtained in a way to minimize the *GBOD* for the plant. These data are listed in the Table 5.

Table 5 Optimal values of *GBOD* and the values of the variables at which they are obtained

Pro-duction demands	H, [h]	<i>GBOD</i> , kg O ₂	Pro-ducts	$x(i)$, %	$B(i)$, [kg]	$NB(i)$	Units assigned
Q-I	H-I	146.943	A	0.633	61.798	89	1, 7, 11
			B	1.071	68.966	87	2, 3, 4, 5, 8
Q-II	H-I	178.096	A	0.93	112.904	62	2, 3, 5, 7, 8, 9
			B	1.131	78.683	89	1, 6, 10, 11
Q-I	H-II	146.943	A	0.633	61.798	89	1, 7, 11
			B	1.071	68.966	87	2, 3, 4, 5, 8
Q-II	H-II	178.058	A	1.055	70.707	99	2, 5, 9, 11
			B	1.079	104.482	67	1, 4, 6, 7, 8, 10

The batch sizes and number of batches subject to processing into the horizons H-I and H-II are shown in the same table. In Table 5 it could be seen that for the production demands Q-I in both horizons, equal values of *GBOD* are obtained, while for the demands Q-II, which are more strained, the optimal values of *GBOD* are diverse and are reached at different values of the independent variables. In real practice, the

different milkfat content effects on the raw material consumption are what could be taken into account. But in the case considered here it is not accounted. For illustration this affect is shown in *Table 6*.

Table 6 Raw material consumption for processing demand Q-II in horizons H-I and H-II

Pro-duction demand	Pro-ducts	Amount , [kg]	$x(i)$, %	Skimmed milk consumption in H-I [kg]	$x(i)$, %	Skimmed milk consumption in H-II [kg]
Q-II,	A	7000	0.93	$2.818 \cdot 10^4$	1.055	$2.812 \cdot 10^4$
	B	7000	1.131	$3.034 \cdot 10^4$	1.069	$3.046 \cdot 10^4$

Finally, the Local *BOD* values for the processed pollutants are listed in *Table 7* to illustrate the weight of the inherent losses into the “generated” *GBOD*. They constitute approximately 28.7% of *OD* generated in accomplishing production demands Q-I and Q-II.

Table 7 Values of Local BODs for the pollutants from the tasks for demands Q-I, and Q- II accomplished in horizons H-I and H-II

Products A and B, kg	Routine source	<i>BOD [kg O₂]</i>		
		Pasteurized milk	Whey	Curds
Q-I, H-I and H-II				
$x_i=0.633\%$ $x_i=1.071\%$	Task 1	63.822		
	Task 2		18.213	
	Task 3		40.889	24.019
Q-II, H-I [h]				
$x_i=0.93\%$ $x_i=1.131\%$	Task 1	77.251		
	Task 2		21.998	
	Task 3		49.788	29.059
Q-II, H-II [h]				
$x_i=1.055\%$ $x_i=1.079\%$	Task 1	77.326		
	Task 2		22.028	
	Task 3		49.783	28.925

Conclusions

The general problem for determining the minimum environmental impact for compatible products manufacturing in multipurpose batch plants is considered in this study. It presents an evolving of the process/units assignment level, proposed in the first part [4] approach, based on the Fourier transformation use, for mathematical descriptions of the waste emissions from routine sources appearing into the horizon cyclically. The problem takes into accounts both the used materials compositions and the constructed production routes, which are set as independent variables. The formulated sets of constraints follow for feasibility and compatibility of the chosen production routes and justify the accomplishment of the production demands into the determined horizon. Global or Local Environmental Impact Assessments are used as the objective function, and for their definition the mathematical descriptions of pollutants from cyclic batch routine sources are used.

An example from the dairy industry is considered to illustrate the possibilities of the proposed system oriented approach for modeling the environmental impact of the multipurpose batch plants. Simultaneous processing of two types of curds - low fat and high fat is regarded. The biological oxygen demand is used to assess the dairy environmental impact. Both the BOD generated due to the amount and composition of the processed milk and the one due to inherent losses, are taken into account in the formulated problem.

The most appropriate milkfat content of the skimmed milk used in products manufacturing and respective process-units assignments (production routes) is determined to result in minimum values of the “processed” Global BOD from the entire plant. The considerable contribution of inherent losses into the GBOD is illustrated too.

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SYMBOLS

B – batch sizes [kg];
C – waste mass concentration [kg/kg]
CC%, – casein content in the curds;
CL% – inherent loss of curds;
FC% – fat content in the curds;
H – time horizon [h] ;
 H-I, H-II – names of horizons used in the example;
I – number of products;
ID – units/tasks identification matrix
L – number of production tasks;
MC% – casein content in the skim-milk;
MI – amount of material input to the task [kg];
ML% –lactose content in the skim-milk;
MO – amount of material output from the task [kg];
MP% –protein content in the skim-milk;
N – number of key components in used materials
NB – number of batches;
P – number of plant units
Q – production demand [kg];
 Q-I, Q-II – names of production demands used in the example;
R – waste processed in the task [kg];
RC – recovery factor for casein;
RF – recovery factor for milkfat;
RS – solids recovery factor;
SC% – solids content in curds;
T – processing time [h];
TC – cycle time [h];
Ts – starting time of task with regard to the cycle beginning;
V – apparatus volume [m³];
WL% – inherent loss of whey;
X – vector of the key components;
m – mass of waste processed from the production task per 1 kilogram target product [kg/kg];
s – size factor [m³/kg];
t – time [h];
x –particular key component composition in used materials continuous variables, (milkfat content for the example)

SUBSCRIPTS

i – product; *j*
j, j' – input output flows of the task;
k – a series order;
l – production task;
n – key component;
p – plant unit;
w – waste

GREEK LETTERS

μ – the standard limit value for the pollutant;
 ν – volume of input to the task flow [m³];
 ζ – binary variables presenting the process-units assignments

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