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QUASI-POLYNOMIAL REPRESENTATION-BASED CONTROL OF MECHANICAL SYSTEMS

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A simple kinematic model of a differential steering mobile robot is investigated using a nonlinear technique based on the quasi-polynomial representation of the dynamical model. Dynamical systems can be embedded in the generalized Lotka-Volterrs (or quasi-polynomial) form under mild conditions. Quasi-polynomial systems are good candidates for a general nonlinear system representation, since their global stability analysis is equivalent to the feasibility of a linear matrix inequality. The stabilizing quasi-polynomial state feedback controller design problem is equivalent to the feasibility of a bilinear matrix inequality. The classical stabilizing state feedback problem for quasi-polynomial systems was extended with the ability of tracking time-dependent reference signals. It is shown that the stabilizing quasi-polynomial controller design is equivalent to a bilinear matrix inequality. The results are applied to the model of the differential steering mobile robot. The goal reaching quasi-polynomial controller is shown to be a special kind of proportional state feedback.

Keywords: quasi-polynomial, robotics, differential drive robot, control systems, kinematics, Lotka-Volterra system

Introduction

Trajectory and goal tracking of mobile robots is an intensively studied field of modern robotics as well as modern control theory. Several papers deal with an adaptive output feedback approach [1]. On the other hand some groups try to apply neural network-based methods for the task [2]. Another direction is to describe the problem as an optimal control problem and apply optimal control results for it [3]. The class of quasi-polynomial (QP) systems plays an important role in the theory of nonlinear dynamical systems because nonlinear systems with smooth nonlinearities can be transformed into quasi-polynomial form Ref.[4]. This means, that any applicable method for quasi-polynomial systems can be regarded as a general technique for nonlinear systems [5]. The aim of this work is to widen the applicability of the quasi-polynomial representation-based methods to the class of mechanical systems, more precisely to mobile robots. The goal reaching problem of a differential steering mobile robot is reformulated as a globally stabilizing feedback design problem in the quasi-polynomial framework.

Basic Notions

Differential Drive Mobile Robot Kinematics

The chosen mechanical system is the kinematic model of a two wheeled differential drive mobile robot *Eq.(1)*. The model deals with the geometric relationships that govern the system. It calculates the motion without considering the affecting forces. The system's states are the Cartesian coordinates x, and y and the orientation θ of the mobile robot. The basic kinematic model of the differential drive robot is given by

$$\begin{aligned} \dot{\theta} &= \frac{r}{2a}(\omega_{\mathrm{left}} - \omega_{\mathrm{right}}) \\ \dot{x} &= \frac{r}{2}\cos(\theta)(\omega_{\mathrm{left}} + \omega_{\mathrm{right}}) \\ \dot{y} &= \frac{r}{2}\sin(\theta)(\omega_{\mathrm{left}} + \omega_{\mathrm{right}}), \end{aligned} \tag{1}$$

where a is half the shaft's diameter, r is the radius of the wheels and ω is the angular velocity of the right or left wheel. To order the robot to reach a specific goal, it is acceptable to design a proportional controller to govern the expected trajectory *Eq.(2)* [6] . In this case, the model is modified to calculate the state errors between the ordered and the present value. The new error model with proportional gain is

$$\begin{aligned} \dot{x} &= K_{\mathrm{v}}(e_{\mathrm{dis}}\cos(\theta) - x(t)) \\ \dot{y} &= K_{\mathrm{v}}(e_{\mathrm{dis}}\sin(\theta) - y(t)) \\ \dot{\theta} &= K_{\mathrm{h}}(e_{\mathrm{ang}} - \theta(t)) \\ e_{\mathrm{dis}} &= \sqrt{(x_{\mathrm{g}} - x(t))^2 + (y_{\mathrm{g}} - y(t))^2} \\ e_{\mathrm{ang}} &= \arctan\left(2\frac{y_{\mathrm{g}} - y(t)}{x_{\mathrm{g}} - x(t)}\right), \end{aligned} \tag{2}$$

where K_{v} is the velocity control gain, K_{h} is the rotational velocity control gain, e_{dis} is the distance error, e_{ang} is the angular error, and $x_{\rm g}$, $y_{\rm g}$ are the Cartesian coordinates to reach.

Quasi-Polynomial Representation of Nonlinear Systems

Some basic notions of quasi-polynomial and Lotka-Volterra systems are summarised in this section.

Generalised Lotka-Volterra Form

Representing an ODE in generalised Lotka-Volterra (GLV) form can increase the structural simplicity in exchange for increasing its dimension. The GLV or quasi-polynomial (QP) form:

$$\begin{aligned} \dot{x}_i &= x_i \left(\lambda_i + \sum_{i=1}^{m} A_{ij} \cdot \prod_{k=1}^{n} x_k^{B_{ij}} \right), \\ i &= 1, \ldots, n, \\ m &\geq n \end{aligned} \tag{3}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are $n \times m$, $m \times n$ real matrices, and $\boldsymbol{\lambda} \in \mathbb{R}^n$ is a vector. The set of non-linear ODEs can be embedded into QP form if it meets two requirements: (i) The non-linear ODEs should follow this form:

$$\begin{aligned} \dot{x}_s &= \sum_{i_{s1} \ldots i_{sn}, j_s} a_{i_{s1} \ldots i_{sn}, j_s} x_1^{i_{s1}} \ldots x_n^{i_{sn}} f(\bar{x})^{j_s}, \\ x_s(t_0) &= x_s^0, \\ s &= 1 \ldots n \end{aligned} \tag{4}$$

where $a_{i_{s1} \ldots i_{sn}, j_s} \epsilon R$, $s = 1 \ldots n$, and $f(\bar{x})$ is some scalar function which cannot be reduced to quasy-monomial form. (ii) The partial derivatives of the system *Eq.(4)* should fulfil:

$$\frac{\partial f}{\partial x_s} = \sum_{e_{s1} \ldots e_{sn}, e_s} b_{e_{s1} \ldots e_{sn}, e_s} x_1^{e_{s1}} \ldots x_n^{e_{sn}} f(\bar{x})^{e_s}, \tag{5}$$

where $b_{e_{s1} \ldots e_{sn}, e_s} \epsilon R$, $s = 1 \ldots n$. By embedding, we introduce the new auxiliary variable:

$$\begin{aligned} y &= f^q \prod_{s=1}^{n} x^p s_s. \\ q &\neq 0 \end{aligned} \tag{6}$$

Differentiating the new, substituted equations we get the QP representation of the original equation *Eq.(4)*:

$$\begin{aligned} \dot{x}_s &= x_s \left[\sum_{i_{s1} \ldots i_{sn}, j_s} \left(a_{i_{s1} \ldots i_{sn} j_s} y^{j_s/q} \right. \right. \\ & \left. \left. \prod_{k=1}^{n} x^{i_{sk} - \delta_{sk} - j_s p_k / q} \right) \right], \\ s &= 1 \ldots n \end{aligned} \tag{7}$$

where $\delta_{sk} = 1$ if $s = k$ and 0 otherwise. A new additional dimension appears as the ODE of the new variable y:

$$\begin{aligned} \dot{y} &= y \Bigg[\sum_{i_{s1} \ldots i_{sn}, j_s} \Bigg(p_s x_s^{-1} \dot{x}_s + \\ &+ \sum_{\substack{i_{s\alpha}, j_s \\ e_{s\alpha}, e_s}} a_{i_{s\alpha}, j_s} b_{i_{s\alpha}, j_s} q y^{(e_s + j_s - 1)/q} \times \\ &\times \prod_{k=1}^{n} x_k^{i_{sk} + e_{sk} + (1 - e_s - j_s) p_k} \Bigg) \Bigg]. \\ \alpha &= 1 \ldots n \end{aligned} \tag{8}$$

It is important to mention that the new ODE is not unique because we can choose the parameters p_s and q.

The quasi-monomial transformation is defined as:

$$\begin{aligned} x_i &= \prod_{k=1}^{n} x_k^{C_{ik}}, \\ i &= 1 \ldots n \end{aligned} \tag{9}$$

where C is an arbitrary invertible matrix. The matrices of GLV can be modified as $\hat{B} = B \cdot C$, $\hat{A} = C^{-1} \cdot A$, and $\hat{\lambda} = C^{-1} \cdot \lambda$, and the transformed set is also in GLV form.

Lotka-Volterra Models

The above family of models is split into classes of equivalence according to the values of the products $\boldsymbol{M} = \boldsymbol{B}\,\boldsymbol{A}$ and $\boldsymbol{N} = \boldsymbol{B}\,\boldsymbol{L}$. The *Lotka-Volterra form* gives the representative elements of these classes of equivalence. If $\text{rank}(\boldsymbol{B}) = n$, then the set of ODEs in *Eq.(3)* can be embedded into the following *m*-dimensional set of equations, the so-called Lotka-Volterra model:

$$\dot{z}_j = z_j \left(N_j + \sum_{i=1}^{m} M_{ji} z_i \right), \quad j = 1, \ldots, m \tag{10}$$

where

$$\boldsymbol{M} = \boldsymbol{B}\,\boldsymbol{A}, \quad \boldsymbol{N} = \boldsymbol{B}\,\boldsymbol{L},$$

and each z_j represents a so-called *quasi-monomial*:

$$z_j = \prod_{k=1}^{n} y_k^{B_{jk}}, \qquad j = 1, \ldots, m. \tag{11}$$

Input-Affine QP Model

The well known input-affine model of nonlinear systems is given in the following state space model:

$$\dot{x} = \boldsymbol{f}(\boldsymbol{x}) + \sum_{j=1}^{p} \boldsymbol{g_j}(\boldsymbol{x}) u_j$$

where $\boldsymbol{f} \in \mathbb{R}^n \to \mathbb{R}^n$, and $\boldsymbol{g_j} \in \mathbb{R}^n \to \mathbb{R}^n$ are QP functions and the input variable $\boldsymbol{u}$ is p-dimensional. In

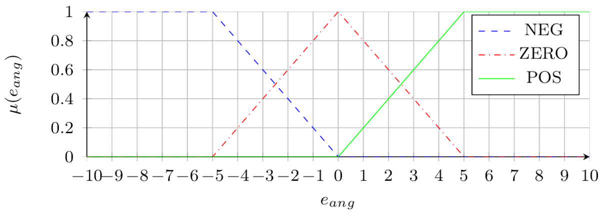

Figure 1: Membership functions for angular error

this case we need a QP form with the matrices from the GLV form ODE set:

$$\begin{aligned}\dot{x}_i &= x_i\left(\lambda_i+\sum_{j=1}^{m}A'_{ij}\prod_{k=1}^{n}x_k^{B_{jk}}\right)+\\ &+\sum_{l=1}^{p}x_i\left(\mu_{li}+\sum_{j=1}^{m}C_{lij}\prod_{k=1}^{n}x_k^{B_{jk}}\right)u_l.\\ i &= 1\dots n\end{aligned} \tag{12}$$

It can be proven that if $u = H(x)$ state feedback is in QP form, the closed-loop system remains in QP form as well, but the quasi-monomials of the system will be greater, than the system's without the feedback.

Global Stability Analysis

Global equilibrium points can be obtained by finding a Lyapunov function $V(\cdot)$. For LV systems there is a well known Lyapunov function family:

$$\begin{aligned}V(\boldsymbol{z}) &= \sum_{i=1}^{m}c_i\left(z_i-z_i^*-\ln\frac{z_i}{z_i^*}\right),\\ &\quad c_i\geq 0, i=1\dots m\end{aligned} \tag{13}$$

and the time derivative:

$$\begin{aligned}\dot{V}_z &= \frac{\partial V(\boldsymbol{z})}{\partial\boldsymbol{z}}\cdot\frac{\partial\boldsymbol{z}}{\partial t}=\\ &= \frac{1}{2}(\boldsymbol{z}-\boldsymbol{z}^*)(\boldsymbol{C}\,\boldsymbol{M}+\boldsymbol{M}^T\,\boldsymbol{C})(\boldsymbol{z}-\boldsymbol{z}^*),\end{aligned} \tag{14}$$

where $\boldsymbol{z^*} = (z_1^*\dots z_m^*)^T$ is the unique positive equilibrium point, $\boldsymbol{C} = \mathrm{diag}(c_1,\dots,c_m)$ and $\boldsymbol{M}$ is the invariant coefficient matrix of the LV form. If $\boldsymbol{C}\,\boldsymbol{M}^T+\boldsymbol{M}\,\boldsymbol{C}$ is negative semi-definite then $\boldsymbol{z}^*$ is stable and if negative definite then $\boldsymbol{z}^*$ is asymptotically stable. The global stability analysis is thus equivalent to the linear matrix inequality

$$\begin{aligned}\boldsymbol{C}\,\boldsymbol{M}^T+\boldsymbol{M}\,\boldsymbol{C} &\leq 0\\ \boldsymbol{C} &> 0.\end{aligned} \tag{15}$$

The presented Lyapunov function *Eq.(13)* can be extended to GLV systems by embedding the system using the LV coefficient matrix $\boldsymbol{M} = \boldsymbol{B}\,\boldsymbol{A}$. It is necessary to solve the LMI system *Eq.(15)* for the stability analysis of the QP and LV system:

$$\begin{pmatrix}\boldsymbol{C} & 0\\ 0 & -\boldsymbol{C}\,\boldsymbol{M}^T-\boldsymbol{M}\,\boldsymbol{C}\end{pmatrix}>0 \tag{16}$$

Controller Design in Quasi-Polynomial Representation

The globally stabilizing QP state feedback design problem for QP systems can be formulated as follows [7]. Consider arbitrary quasi-polynomial inputs in the form:

$$u_l=\sum_{i=1}^{r}k_{il}\hat{q}_i,\quad l=1\dots,p \tag{17}$$

where $\hat{q}_i = \hat{q}_i(y_1,\dots,y_n)$, $i = 1,\dots,r$ are arbitrary quasi-monomial functions of the state variables of the system and k_{il} is the constant gain of the quasi-monomial function $\hat{q}_i$ in the l-th input u_l. The closed-loop system will also be a QP system furthermore, the closed-loop LV coefficient matrix $\hat{\boldsymbol{M}}$ can also be expressed in the form

$$\hat{\boldsymbol{M}}=\hat{\boldsymbol{B}}\,\hat{\boldsymbol{A}}=\boldsymbol{M_0}+\sum_{l=1}^{p}\sum_{i=1}^{r}k_{il}\boldsymbol{M_{il}}. \tag{18}$$

Then the global stability analysis of the closed-loop system with unknown feedback gains k_{il} leads to the following *bilinear matrix inequality*

$$\begin{aligned}&\hat{\boldsymbol{M}}^T\boldsymbol{C}+\boldsymbol{C}\hat{\boldsymbol{M}}=\boldsymbol{M_0}^T\boldsymbol{C}+\boldsymbol{C}\boldsymbol{M_0}+\\ &\qquad\sum_{l=1}^{p}\sum_{i=1}^{r}k_{il}\left(\boldsymbol{M_{il}}^T\boldsymbol{C}+\boldsymbol{C}\boldsymbol{M_{il}}\right)\leq 0.\end{aligned} \tag{19}$$

The variables of the BMI are the $p\times r$ k_{il} feedback gain parameters and the c_j, $j = 1,..,m$ parameters of the Lyapunov function. If the BMI above is feasible then there exists a globally stabilizing feedback with the selected structure.

Quasi-Polynomial Control of a Differential Drive Mobile Robot

Quasi-Polynomial Representation of the Kinematic Model

With the given differential drive robot model *Eq.(1)* and the explained error model *Eq.(2)* the QP representation can be built. The fist step as mentioned, is to find the new auxiliary variables, that help to eliminate the non-QP expressions. The newly chosen auxiliary variables are:

$$\begin{aligned}\alpha &= \cos(\theta),\\ \beta &= \sin(\theta),\\ \gamma &= \arctan\left(2\frac{y_g-y}{x_g-x}\right),\\ \delta &= (x_g-x)^2+(y_g-y)^2,\text{ and}\\ \epsilon &= \sqrt{\delta}+(x_g-x).\end{aligned} \tag{20}$$

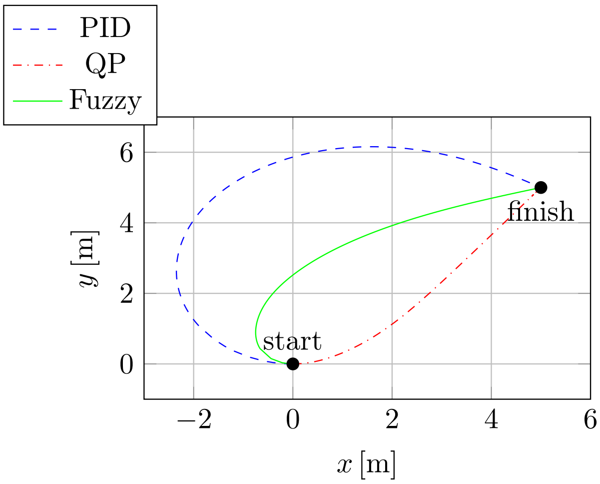

Figure 2: Comparison of different controllers

By substituting them into the original system *Eq.(2)* and performing the differentiation of the new equations *Eq.(20)*:

$$\begin{aligned} \dot{x} &= x\left(\frac{K_\mathrm{v}\sqrt{\delta}\alpha}{x} - K_\mathrm{v}\right) \\ \dot{y} &= y\left(\frac{K_\mathrm{v}\sqrt{\delta}\beta}{y} - K_\mathrm{v}\right) \\ \dot{\theta} &= \theta\left(\frac{K_\mathrm{h}\gamma}{\theta} - K_\mathrm{h}\right) \\ \dot{\alpha} &= \alpha\left(-\frac{\beta}{\alpha}\right) \\ \dot{\beta} &= \beta\left(\frac{\alpha}{\beta}\right) \end{aligned} \tag{21}$$

$$\begin{aligned} \dot{\gamma} = \gamma\cdot \Bigg(& -\frac{2K_\mathrm{v}\beta\sqrt{\delta}}{\gamma\epsilon} + \frac{K_\mathrm{v}y}{\gamma\epsilon} - \frac{K_\mathrm{v}x_\mathrm{g}\beta}{\gamma\epsilon} + \frac{K_\mathrm{v}x\beta}{\gamma\epsilon} \\ & + \frac{K_\mathrm{v}x_\mathrm{g}y}{\gamma\epsilon\sqrt{\delta}} - \frac{K_\mathrm{v}xy}{\gamma\epsilon\sqrt{\delta}} + \frac{K_\mathrm{v}y_\mathrm{g}\alpha}{\gamma\epsilon} - \frac{K_\mathrm{v}y\alpha}{\gamma\epsilon} \\ & - \frac{K_\mathrm{v}y_\mathrm{g}x}{\gamma\epsilon\sqrt{\delta}} + \frac{K_\mathrm{v}xy}{\gamma\epsilon\sqrt{\delta}}\Bigg) \end{aligned} \tag{22}$$

$$\begin{aligned} \dot{\delta} = \delta \Bigg(& -\frac{2K_\mathrm{v}x_\mathrm{g}\alpha}{\sqrt{\delta}} + \frac{2K_\mathrm{v}x_\mathrm{g}x}{\delta} + \frac{2K_\mathrm{v}x\alpha}{\sqrt{\delta}} - \frac{2K_\mathrm{v}x^2}{\delta} \\ & - \frac{2K_\mathrm{v}y_\mathrm{g}\beta}{\sqrt{\delta}} + \frac{2K_\mathrm{v}y_\mathrm{g}y}{\delta} + \frac{2K_\mathrm{v}y\beta}{\sqrt{\delta}} - \frac{2K_\mathrm{v}y^2}{\delta}\Bigg) \end{aligned} \tag{23}$$

$$\begin{aligned} \dot{\epsilon} = \epsilon \Bigg(& -\frac{K_\mathrm{v}x_\mathrm{g}\alpha}{\epsilon} + \frac{K_\mathrm{v}x\alpha}{\epsilon} + \frac{K_\mathrm{v}x_\mathrm{g}x}{\epsilon\sqrt{\delta}} - \frac{K_\mathrm{v}x^2}{\epsilon\sqrt{\delta}} \\ & - \frac{K_\mathrm{v}y_\mathrm{g}\beta}{\epsilon} + \frac{K_\mathrm{v}y\beta}{\epsilon} + \frac{K_\mathrm{v}y_\mathrm{g}y}{\epsilon\sqrt{\delta}} - \frac{K_\mathrm{v}y^2}{\epsilon\sqrt{\delta}} \\ & + \frac{K_\mathrm{v}\alpha}{\epsilon} - \frac{K_\mathrm{v}x}{\epsilon\sqrt{\delta}}\Bigg) \end{aligned} \tag{24}$$

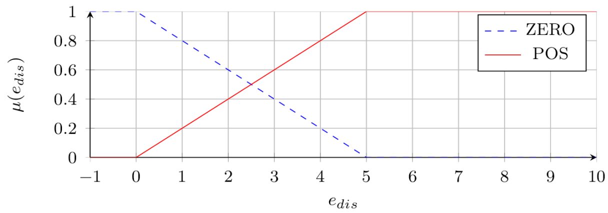

Figure 3: Membership functions for distance error

The closed-loop QP system *Eq.(21-24)* has 29 quasi-monomials.

$$\begin{array}{rlll} x: & \alpha\sqrt{\delta}x^{-1} & & \\ y: & \beta\sqrt{\delta}y^{-1} & & \\ \theta: & \gamma\theta^{-1} & & \\ \alpha: & \beta\alpha^{-1} & & \\ \beta: & \alpha\beta^{-1} & & \\ \gamma: & \beta\sqrt{\delta}\gamma^{-1}\epsilon^{-1}, & y\gamma^{-1}\epsilon^{-1}, & \beta\gamma^{-1}\epsilon^{-1}, \\ & x\beta\gamma^{-1}\epsilon^{-1}, & y\gamma^{-1}\delta^{-1/2}\epsilon^{-1}, & \alpha\gamma^{-1}\epsilon^{-1}, \\ & y\alpha\gamma^{-1}\epsilon^{-1}, & x\gamma^{-1}\delta^{-1/2}\epsilon^{-1} & \\ \delta: & \alpha\delta^{-1/2}, & x\delta^{-1}, & x\alpha\delta^{-1/2}, \\ & x^2\delta^{-1}, & \beta\delta^{-1/2}, & y\delta^{-1}, \\ & y\beta\delta^{-1/2}, & y^2\delta^{-1} & \\ \epsilon: & \alpha\epsilon^{-1}, & x\alpha\epsilon^{-1}, & x\epsilon^{-1}\delta^{-1/2}, \\ & x^2\epsilon^{-1}\delta^{-1/2}, & \beta\epsilon^{-1}, & y\beta\epsilon^{-1}, \\ & y\epsilon^{-1}\delta^{-1/2}, & y^2\epsilon^{-1}\delta^{-1/2}, & \end{array} \tag{25}$$

The QP system *Eq.(21-24)* can obtain the GLV form invariants with the help of monomials.

These matrices are: $\boldsymbol{A}$ the coefficient matrix, $\boldsymbol{B}$ the exponential matrix and $\boldsymbol{\lambda}$ the constant's matrix. The invariant matrix product $\boldsymbol{B}\,\boldsymbol{A}$ results in the LV coefficient matrix $\boldsymbol{M}$, which is necessary for the global stability analysis.

Controller Design

As the original model *Eq.(2)* already contains the proportional controller parameters K_h and K_v, they naturally appear in the QP and Lotka-Volterra forms, respectively.

The closed-loop Lotka-Volterra coefficient matrix $\boldsymbol{M}$ contains the proportional gains in a linear manner *Eq.(26)* [7]

$$\boldsymbol{M} = \boldsymbol{M}_0 + K_\mathrm{h}\,\boldsymbol{M}_\mathrm{h} + K_\mathrm{v}\,\boldsymbol{M}_\mathrm{v}, \tag{26}$$

where $\boldsymbol{M} \in \mathbb{R}^{29\times 29}$. This means, that the globally stabilizing feedback design BMI can be formulated, and checked for feasibility.

The controller design is practically solved as a control-Lyapunov function-based state feedback. The prescribed Lyapunov function parameters are $c_{1,2,3} = 0.1$, $c_{4,5} = 1$ and $c_{6-29} = 0$. The control gains are obtained by solving an LMI version of the feedback design BMI by substituting $\boldsymbol{C}$ into *Eq.(19)*. The obtained values are $K_\mathrm{h} = 2.78$ and $K_\mathrm{v} = 0.8$.

Table 1: Rules for the fuzzy controller

$\wedge$			$\wedge$	
e_{ang}	e_{dis}		ω_{left}	ω_{right}
ZERO	ZERO		ZERO	ZERO
POS	ZERO		POS	NEG
	ZERO	$\Rightarrow$	NEG	POS
ZERO	POS		POS	POS
POS	POS		POS	ZERO
	POS		ZERO	POS

Comparison of Results for Different Controller Types

The first and most basic method was the proportional controller approach. This was explained in the second chapter. This is a more systematic approach, but with the increasing non-linearities it is becoming a more difficult method to solve. *Fig.3* shows that the robot moves on a significantly wider curve compared to the other two designs.

Secondly a fuzzy controller was designed to compare the original design to. This is a much more intuitive approach, the physical nonlinearities were easily implementable with the cost of inaccuracy. This can be seen in *Fig.3*, where the robot travels on a much smaller curve. *Table 1* shows the definition of the rules. The membership functions follow simple triangular shapes according to the acceptable rate of angular and velocity changes. The QP controller performance can be seen in *Fig.3*. where a classical PID controller and a fuzzy logic controller are also shown. It is apparent, that the QP controller performs well by means of the length of the trajectory.

Conclusions

A quasi-polynomial representation-based nonlinear control design method has been applied to the kinematics of a differential steering mobile robot in this work. It has been shown, that the quasi-polynomial model of the mobile robot's kinematics extended with the tracking error dynamics has 29 quasi-monomials, i.e. the sizes of matrices appearing in the bilinear matrix inequality that should be solved for a globally stabilizing feedback controller are reasonably great. The resulting controller and the trajectory were compared to the trajectories of a reference PID controller and a fuzzy logic controller. It can be seen that the controller performs satisfactorily. Possible future directions include the application of the LMI and /or BMI cost function to introduce some optimality measure to the problem to be minimised. A next step would be the application of a time dependent goal position (i.e. a trajectory).

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