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MODELING AND PARAMETER SENSITIVITY ANALYSIS OF A SYNCHRONOUS MOTOR

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A simple dynamic model of a synchronous motor is developed in this paper based on first engineering principles that describe the mechanical phenomena together with the electrical model. The constructed state space model consists of nonlinear state equations and bi-linear output equations. The developed model has been verified under the usual regulated operating conditions when the speed and the torque are controlled, the manipulated input is the network voltage and the exciter voltage. The effect of load on the controlled synchronous motor has been analyzed by simulation using PI controllers. Model parameter sensitivity analysis has been applied to determine the model parameters to be estimated.

Keywords: synchronous machine, dynamic state space model, sensitivity analysis, parameter estimation

Introduction

Classical synchronous motors are widely used machines when constant speed is necessary. The speed control of synchronous machines is a difficult problem since the motor speed is a linear function of the network frequency. For the control of the synchronous motor (SM) we have to use an inverter which generates the three phase sinusoidal electrical network and a DC power supply which provides the exciter voltage.

The final aim of our study is to design a controller that regulates the speed and the torque of the synchronous motor.

Because of the specialties and great practical importance of synchronous machines in industry, their modeling for control purposes is well investigated in the literature. Besides of the basic textbooks (see e.g. [1, 2]), there are papers that describe the modelling and use the developed models for the design of various controllers.

The aim of this paper is to perform the parameter sensitivity analysis of a simple dynamic model of a synchronous motor. The result of this analysis will be the next step of the parameter estimation.

The model of the synchronous motor

Modelling assumptions

For constructing the synchronous motor model, let us make the following assumptions:

- a symmetrical tri-phase stator winding system is assumed,
- one field winding is considered to be in the machine,
- all the windings are magnetically coupled,
- the flux linkage of the windings is a function of rotor position,
- the copper losses and the slots in the machine are neglected,
- the spatial distribution of stator fluxes and apertures wave are considered to be sinusoidal,
- the stator and rotor permeability are assumed to be infinite.
- It is also assumed that all the losses due to wiring, saturation and slots can be neglected.

The four windings (three stators and one rotor) are magnetically coupled. Since the magnetic coupling between the windings is a function of the rotor position, the flux linkage of the windings is also a function of the rotor position. The actual terminal voltage v of the windings can be written in the form

$$v = \pm\sum_{j=1}^{J}(r_j \cdot i_j) \pm \sum_{j=1}^{J}(\dot{\lambda}_j) \tag{1}$$

where i_j are the currents, r_j are the winding resistances, and λ_j are the flux linkages.

The positive directions of the stator currents point in the synchronous motor terminals.

Thereafter, the two stator electromagnetic fields, both travelling at rotor speed, were identified by decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field and another phase shifted by 90°. With the above, one can construct an air-gap field with its maximal aligned to the rotor poles (d axis), while the other is aligned to the q axis (between poles). This method is called the Park's transformation [3, 4].

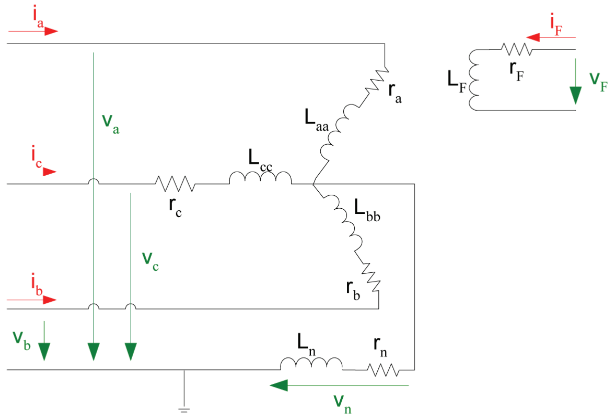

Figure 1: The equivalent circuit of the SM

As a result, the vector voltage equation is:

$$v_{dFq} = R_{RS\omega} i_{dDq} + L\dot{i}_{dFq} \tag{2}$$

with

$$i_{dFq} = \begin{bmatrix} i_d & i_F & i_q \end{bmatrix}^T$$

$$v_{dFq} = \begin{bmatrix} v_d & v_F & v_q \end{bmatrix}^T$$

where v_d and v_q are the direct and the quadratic components of the stator voltage of the synchronous motor, i_d and i_q are the direct and the quadratic components of the stator current, while v_F and i_F are the exciter voltage and current.

Flux linkage equations

The synchronous motor consists of six coupled coils referred to with indices a, b and c are the stator phases coils, F is the filed coil. The linkage equations can be written in the following form:

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \end{bmatrix} \tag{3}$$

where L_{xx} are the strator and rotor mutual inductances.

After applying Park's transformation, the following linkage equations are obtained:

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F \\ 0 & 0 & L_q & 0 \\ 0 & kM_F & 0 & L_F \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \end{bmatrix} \tag{4}$$

Voltage equations

We can write Kirchoff's voltage laws in the following form:

$$v = ri + \dot{\lambda} + v_n \tag{5}$$

$$v_n = -R_n i_{abc} - L_{nm} \dot{i}_{abc} \tag{6}$$

$$\begin{bmatrix} v_{abc} \\ v_F \end{bmatrix} = \begin{bmatrix} R_{abc} & 0 \\ 0 & R_F \end{bmatrix} \begin{bmatrix} i_{abc} \\ i_F \end{bmatrix} + \begin{bmatrix} \dot{\lambda}_{abc} \\ \dot{\lambda}_{abc} \end{bmatrix} + \begin{bmatrix} v_n \\ 0 \end{bmatrix} \tag{7}$$

$$R_{abc} = \begin{bmatrix} r_a & 0 & 0 \\ 0 & r_b & 0 \\ 0 & 0 & r_c \end{bmatrix}$$

$$L_{nm} = \begin{bmatrix} L_n & L_n & L_n \\ L_n & L_n & L_n \\ L_n & L_n & L_n \end{bmatrix} \quad R_n = \begin{bmatrix} r_n & r_n & r_n \\ r_n & r_n & r_n \\ r_n & r_n & r_n \end{bmatrix}$$

where v_n is the neutral voltage and $R_F = r_F$.

Using Park's transformation is replaced with the equations for the d-q voltage components:

$$\begin{bmatrix} v_d \\ v_F \\ v_q \end{bmatrix} = \begin{bmatrix} r_d & 0 & \omega L_q \\ 0 & r_F & 0 \\ -\omega L_d & -\omega k M_F & r_q \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_q \end{bmatrix} + \begin{bmatrix} L_d & kM_F & 0 \\ kM_F & L_F & 0 \\ 0 & 0 & L_q \end{bmatrix} \begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_q \end{bmatrix} \tag{8}$$

The voltage equation in matrix form (8) is:

$$v_{dFq} = R_{RS\omega} i_{dFq} + L\dot{i}_{dFq} \tag{9}$$

The state space model for the currents is obtained by expressing $\dot{i}_{dFq}$ from (11), i.e.

$$\dot{i}_{dFq} = -L^{-1} \cdot R_{RS\omega} \cdot i_{dFq} - L^{-1} \cdot v_{dFq} \tag{10}$$

Power and torque equations

The electrical energy of the SM is a sum of the following mechanical equations.

$$dW_{Electr} = dW_{Mech} + dW_{Field} + dW_{\Omega} \tag{11}$$

Time derivate of the energy equation is the power equation, which represents the energy change:

$$P_{Mech} = P_{Electr} - P_{Field} - P_{\Omega} \tag{12}$$

Torque is obtained from dividing power by angular velocity.

$$T_{Mech} = \lambda_d i_q - \lambda_q i_d \quad (13)$$

$$\omega = \frac{d\theta}{dt} \quad (14)$$

The torque from field enegy is given by the torque equation:

$$T_{Field} = (i_0 \frac{\lambda_0}{d\theta} + i_d \frac{\lambda_d}{d\theta} + i_q \frac{\lambda_q}{d\theta}) \quad (15)$$

Afterwards the torque is:

$$T_{Mech} = T_{Electr} - T_{Field} - T_{Dump} \quad (16)$$

From Newton's law of motion we can write the speed and torque equation:

$$2H\dot{\omega} = T_{electr} - T_{mech} - D\omega \quad (17)$$

where D is a damping constant. We can write the input power in the following equation:

$$P_{electr} = \omega(\lambda_d i_q - \lambda_q i_d) - (i_0 \frac{\lambda_0}{d\theta} + i_d \frac{\lambda_d}{d\theta} + i_q \frac{\lambda_q}{d\theta}) \quad (18)$$
$$- r(i_0^2 + i_d^2 + i_q^2) - 3r_n i_0^2$$

(In balanced condition $i_0 = 0$.) After we can compute the accelerating torque:

$$T_{Acc} = \frac{T_{Electr3}}{3} - T_{Mech} - T_{Dump} =$$
$$T_{Electr} - T_{Mech} - T_{Dump}$$

To compute eletrical torque we sould write the direct and quadratic part of the stator flux from equation (12):

$$\begin{aligned} \lambda_d &= L_d i_d + kM_F i_F \\ \lambda_q &= L_q i_q \end{aligned} \quad (19)$$

After it the write eletrical torque can be expressed:

$$T_{eletr3} = \begin{bmatrix} L_d i_q & kM_F i_q & -L_q i_d \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_q \end{bmatrix} \quad (20)$$

Using $\dot{\omega} = \frac{T_{Acc}}{dt}$ it is possible compute the speed of the synchronous machine. The motion equation is as follows:

$$\dot{\omega} = \begin{bmatrix} \frac{L_d i_q}{3\tau_j} & \frac{kM_F i_q}{3\tau_j} & -\frac{L_q i_d}{3\tau_j} & -\frac{D}{\tau_j} \end{bmatrix} \cdot \quad (21)$$
$$\begin{bmatrix} i_d & i_F & i_q & \omega \end{bmatrix}^T - \frac{T_{Mech}}{\tau_j}$$

The loading angle (δ) of the synchronous motor is

$$\delta = \delta_0 + \int_{t_0}^{t} (\omega - \omega_R) dt \quad (22)$$

that can be differentiated to obtain the time derivative of δ

$$\dot{\delta} = \omega - \omega_R \quad (23)$$

Altogether, there are five state variables: i_d , i_F , i_q , ω and δ. The input variables (i.e. manipulable inputs and disturbances) are T_{mech}, v_F, v_d and v_q. Observe that the state equations are bilinear in the state variables.

The outputs of the model are the speed (ω) of the motor and the loading angle (δ) of the SM.

Model analysis

The state space model (10, 21, 23) has been verified by simulation against engineering intuition using parameter values of a similar machine [5]. After the basic dynamical analysis, eleven parameters have been selected for sensitivity analysis.

Motor parameters

The parameters are described only for phase *a* since the machine is assumed to have symmetrical tri-phase stator windings system. The stator mutual inductances for phase *a* are:

$$\begin{aligned} L_{ab} &= L_{ba} = -M_s - L_m(2(\Theta - \frac{\pi}{6})) \\ L_{bc} &= L_{cb} = -M_s - L_m(2(\Theta - \frac{\pi}{2})) \\ L_{ca} &= L_{ac} = -M_s - L_m(2(\Theta + \frac{5\pi}{6})) \end{aligned} \quad (24)$$

where M_s is a given constant. The phase a stator to rotor mutual inductances are given by (from phase windings to the field windings):

$$\begin{aligned} L_{aF} &= L_{Fa} = M_F \cos(\Theta) \\ L_{bF} &= L_{Fb} = M_F \cos(\Theta - \frac{2\pi}{3}) \\ L_{cF} &= L_{Fc} = M_F \cos(\Theta + \frac{2\pi}{3}) \end{aligned} \quad (25)$$

where M_F is a given constant. Parameters L_d, L_d, L_0 and M_F used by the state space model (10, 21, 23) are defined as:

$$\begin{aligned} L_d &= L_s + M_s + \frac{3}{2} L_m \\ L_q &= L_s + M_s - \frac{3}{2} L_m \\ L_0 &= L_s - 2M_s \\ M_F &= \frac{L_{AF}}{k} \\ k &= \sqrt{\frac{2}{3}} \end{aligned} \quad (26)$$

Using the initial assumption of symmetrical tri-phase stator windings (i.e. $r_a = r_b = r_c = r$) the resistance of stator windings of the machine we denoted by r. Resistance of the rotor exciter is represented by r_F. Parameters L_d and L_q are the direct and quadratic stator inductances, L_F is the stator exciter inductance. D_{const} presents the damping constant, P is the proportional, I is the integrator parameter of the torque PI controller. The parameter values were obtained from the literature [1]:

$$\begin{array}{ll} L_d = 0.176\text{H} & L_{AF} = 0.1302\text{H} \\ L_q = 0.171\text{H} & r = 0.5577\Omega \\ L_F = 0.138\text{H} & r_F = 0.0195\Omega \\ l_d = 0.0126\text{H} & D = 2.004 \\ l_q = 0.0126\text{H} & I = 0.1 \\ l_F = 0.00849\text{H} & P = 0.05 \end{array} \tag{27}$$

Stability analysis

Eleven parameters of the synchronous motor have been selected for sensitivity analysis, and the sensitivity of the state variables has been investigated by Matlab dynamical simulation.

The equilibrium point of the state space model can be obtained from the steady-state version of state equations (10, 21, 23) using the above parameter values. The equilibrium point of the system is:

$$\begin{aligned} \omega &= 1.779712 \\ i_F &= 2.978999 \\ i_d &= \text{-}2.046594 \\ i_q &= 0.375484 \\ P_{in} &= 2.912249 \end{aligned} \tag{28}$$

The state matrix of the state space model (28) has the following numerical value in this equilibrium:

$$\begin{bmatrix} -3.3714\cdot10^{-2} & 8.8667\cdot10^{-4} & -5.8023 & -1.2242 \\ 2.5277\cdot10^{-2} & -2.4983\cdot10^{-3} & 4.3503 & 9.1782\cdot10^{-1} \\ 6.7901\cdot10^{-3} & 1.2970\cdot10^{-3} & 1.1685 & 2.4654\cdot10^{-1} \\ -6.2364 & -4.6031 & 3.5201\cdot10^{-2} & -5.3330\cdot10^{-1} \end{bmatrix} \tag{29}$$

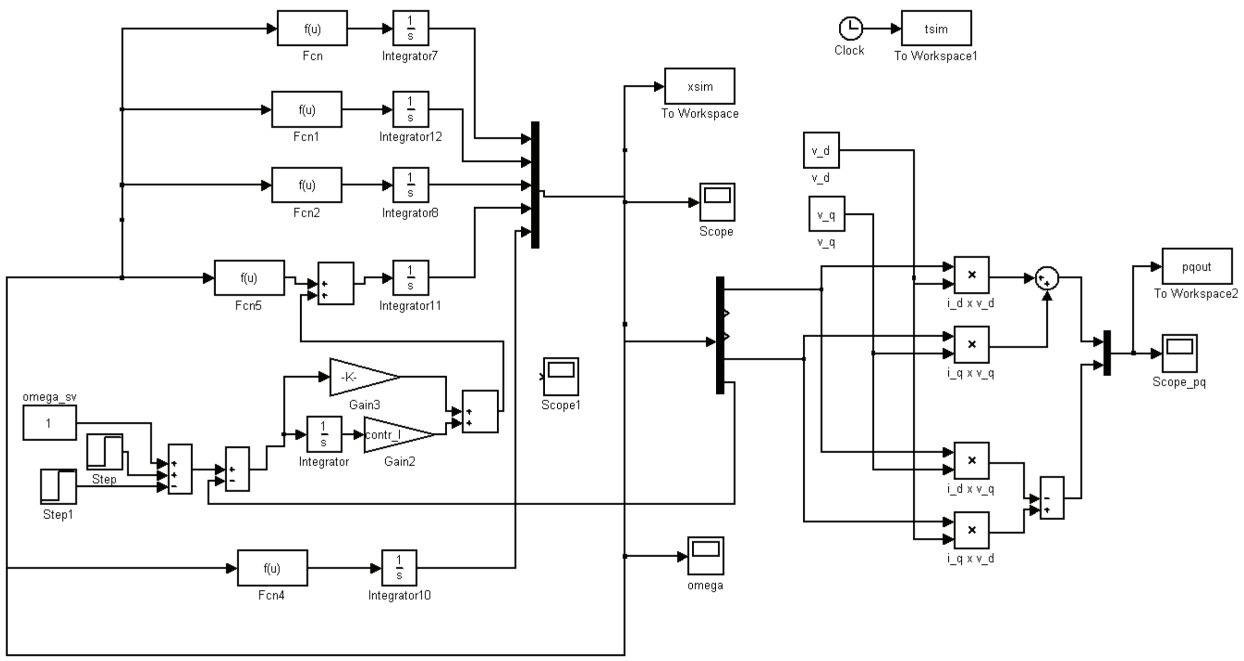

Figure 2: The Matlab Simulink model of the synchronous motor

The eigenvalues of the state matrix are:

$$\begin{aligned} \lambda_{1,2} &= -3.399932\cdot10^{-2} \pm j4.068256 \\ \lambda_3 &= -4.0820501\cdot10^{-3} \\ \lambda_4 &= -1.3064115\cdot10^{-3} \end{aligned} \tag{30}$$

The real parts of the eigenvalues are negative but their magnitudes are small, thus the system is on the boundary of the stability domain.

Parameter sensitivity analysis

PI controller

The applied control method of the synchronous machine is a classical PI controller *(Fig. 2)* that ensures stability of the equilibrium point under small perturbations. The controlled output is the mechanical torque, the manipulated input is the voltage. The proportional parameter of the PI controller of the torque is 0.05 and the integrator time is 0.1 in per units.

Model validation

The dynamical properties of the motor have been investigated. The response of the torque controlled motor has been tested under step-like changes of the exciter voltage. The simulation results are shown in *Figs. 3, 4, 5* and *6* where the quadratic linkage inductance l_q the damping constant D, the stator exciter resistance r_F and the stator resistance r are shown.

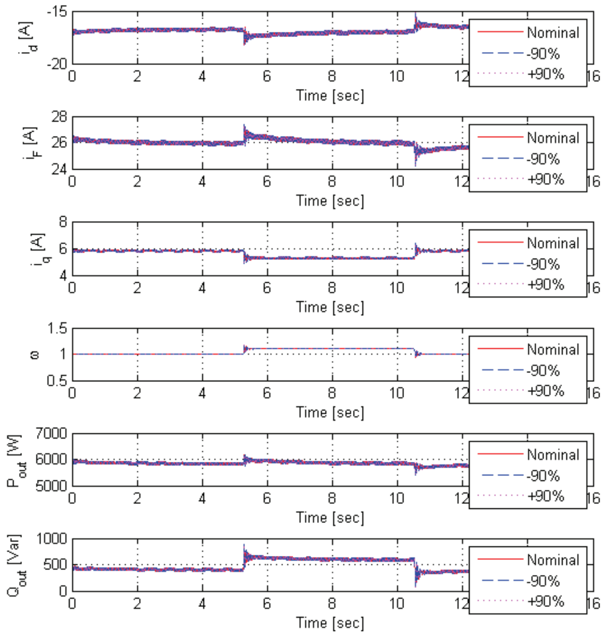

Figure 3: Model responses for the +90% changing of parameter l_q

Sensitivity analysis

The aim of this section is to define parameter groups according to the system's sensitivity on them. Linkage inductances l_d, l_q, l_F are not used by the current model, only by the flux model. As it was expected, the model is insensitive for these parameters. Note, that the linkage inductance parameters are only used for calculating the fluxes of the machine (Fig. 3). Response to the exciter voltage step change of the controlled motor (means the deviation form the steady-state value) Sensitivity of the model to the controller parameters P (proportional) and I (integrator) and the damping constant D has also been investigated.

This is why the output and the steady state value of the system variables do not change even for a considerably large change of D (Fig. 4).

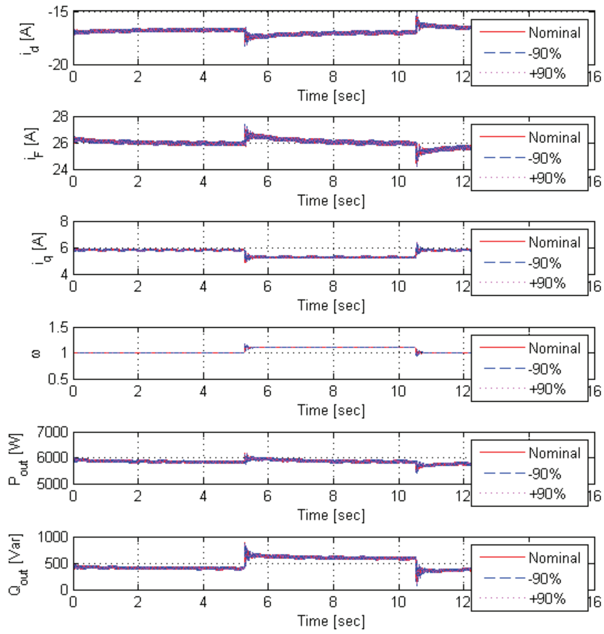

Figure 4: Model responses for the +90% changing of parameter D

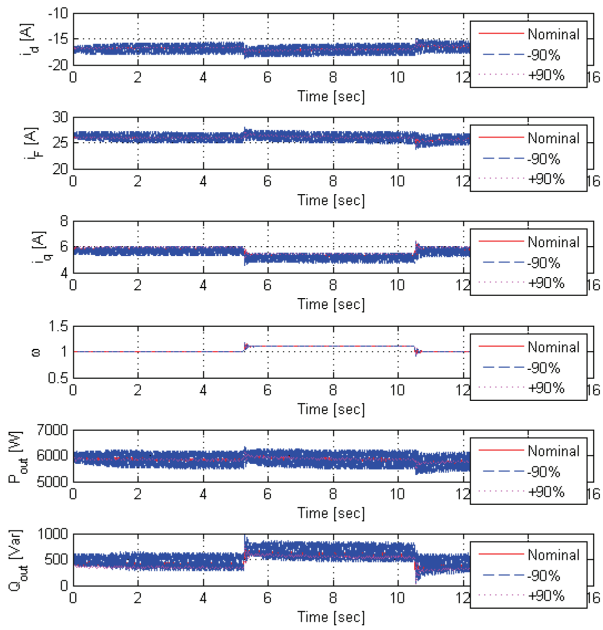

Figure 5: Model responses for the +90% changing of parameter r stator resistance

Not sensitive:
These are the linkage inductances l_d, l_q, l_F and damping constant D. The state space model is insensitive for them, the parameter values cannot be determined from measurement data using any parameter estimation method.

Sensitive:
The stator resistance r, the proportional controller parameter P, the integrator controller parameter I and the stator inductances L_d and L_q.

Critically sensitive:
The rotor exciter resistance r_F and the rotor exciter inductance L_F.

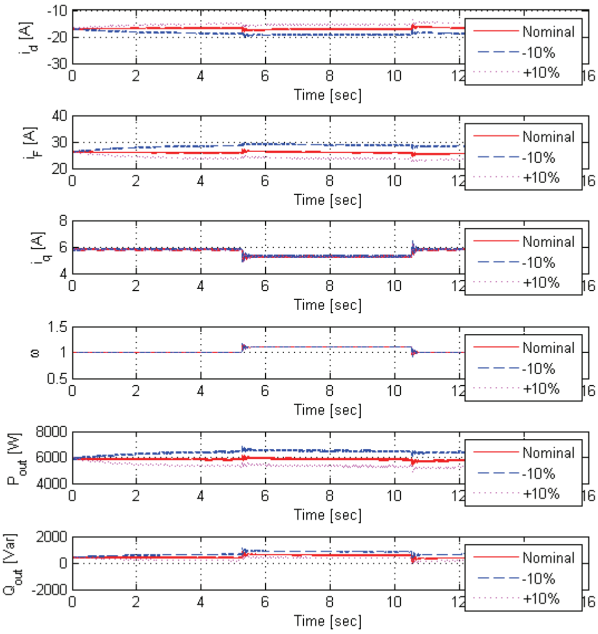

Figure 6: Model responses for the +10% changing of parameter r_F rotor exciter resistance

Conclusions and future works

Based on the results presented here, it is possible to select the candidate parameters for model parameter estimation based on real data that is a further aim of the authors. The four parameters to be estimated in a later work are P (proportional parameter of the controller), L_F (rotor exciter inductance), r_F (rotor exciter resistance), and r (stator resistance).

The final aim of is to develop a simple yet detailed state space model of the induction motor for control purposes which gives us the possibility to develop and analyze different control strategies for the synchronous motor.

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