

DETERMINING OPTIMAL STOCK LEVEL IN MULTI-ECHELON SUPPLY CHAINS

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Inventory control of Multi-echelon supply chains is a widely researched area. In most cases researchers choose analytic methods to analyze such logistics systems. Simulation is a very useful alternative for analyzing supply chain systems and a well-constructed model can provide a better approach and can give more realistic picture of the complex situation. In this article the authors introduce an interactive, configurable simulator to analyze stock levels in a complex supply chain. This new modeling approach (SIMWARE) is capable to simulate complex multi-echelon supply chains where the frequency of stock transfer between the individual levels of the supply chain can be optimized. The authors evaluate different optimization strategies and methods. The introduced SIMWARE method can be used to minimize the environmental impact of the supply chain by minimizing the transportation between the nodes of the supply chain hierarchy. The model provides an optimization methodology where the objective function is the total cost of the supply chain.

Keywords: supply chain, simulation, safety stock, optimization.

Introduction

Supply chains present a complex decision situation for management. Supply chain performance impacts the financial performance of the company therefore it is important to find a way to optimize the performance of the supply chain. Simulation provides a way to get closer to real life situations and use less simplifications and assumptions than you need with analytical solutions.

Our objective is to build a simulator that can use simple building blocks to construct models of complex supply chain networks. Supply chains processes can be simulated using these models, where parameters of Key Performance Indicators are analyzed by sensitivity analysis. The result is a decision support tool flexible enough to handle complex situations and straightforward and simple to use enough for management purposes.

Literature review

The research of inventory management or the investigation of supply chain is widespread in the scientific literature. It is a seriously researched area since the fifties; Simpson [6] was the first one who formulated the serial-line inventory problem. Graves and Willems extend Simpsons work to spanning trees in [2], while in [3] they give a comprehensive review of the previous approaches for safety stock placement, the same as they propose two general approaches and introduce the supply chain configuration problem. There exist good overviews in the

literature for supply chain and inventory management, like in [11], where authors give an overview of different approaches of supply chain modeling, and outline future opportunities. In [9] Lau et al. give an overview of various average inventory level (AIL) expressions and presents two novel expressions which are simpler and more accurate than previous ones. A comprehensive overview can be found in [13], where a simulation model for a real problem with lost sales is presented, where authors consider several types of inventory policies.

The determination of safety stock in an inventory model is one of the key actions in the management, Miranda and Garrido include both cycle and safety stock in the inventory model in [12], and the resulting model in this article has a non-linear objective function. Authors in [4] gives a model for positioning safety stock in a supply chain subject to non-stationary demand, and they show how to extend their former model to find the optimal placement safety stocks under constant time service (CST) policy. Prékopa in [16] gives an improved model for the so called Hungarian inventory control model to find the minimal safety stock level that ensures the continuous production, without disruption.

The bullwhip effect is an important phenomenon in supply chains, authors in [10] show how a supply chain can be modeled and analyzed by colored petri nets (CPN) and CPN tools and they evaluate the bullwhip effect, the surplus of inventory goods, etc. using the beer game as demonstration. More recent research can be found in [1], which shows that an order policy applied to a serial single-product supply chain with four echelons can reduce or amplify the bullwhip effect and inventory oscillation.

Miranda et al. investigate the modeling of a two echelon supply chain system and optimization in two steps [15], while a massive multi-echelon inventory model is presented by Seo [19], where an order risk policy for general multi-echelon system is given, which minimizes the system operation cost. A really complex system is examined in [20], where it is necessary to apply some clustering for similar items, because detailed analysis could become impossible considering each item individually.

The simulation-based approach was published only in the last decade. Jung et al. [7] make a Monte Carlo based sampling from real data, and apply a simulation-optimization framework while looking for managing uncertainty. They use a gradient-based search algorithm, while authors in [8] discuss how to use simulation to describe a five-level inventory system, and optimize this model by genetic algorithm. Schwartz et al. [18] demonstrate the internal model control (IMC) and model predictive control (MPC) algorithms to manage inventory in uncertain production inventory and multi-echelon supply/demand networks. The stability of the supply chain is also a seriously researched area recently, [14] shows that a linear supply chain can be stabilized by the anticipation of the own future inventory and by taking into account the inventories of other suppliers, and Vaughan in [21] presents a linear order point/lot size model that with its robustness can contribute to business process modeling.

A complex instance of inventory model can be found in [5], where orders cross in time considering various distributions for the lead time. Sakaguchi in [17] investigates the dynamic inventory model in which demands are discrete and varying period by period. The author gives an algorithm to solve several examples.

Based on the previous review it is clear that most of the multi-echelon supply chain optimization and analysis are based on analytical approach. Simulation however provides a very good alternative, because it can model real life situations with accuracy, more flexible in terms of input parameters and therefore it is more easy to use in decision support. The simulation results can be analyzed with various statistical methods and numerical optimization algorithms. To analyze complex, especially multi-echelon systems, multi-level simulation models can be used, where the results of optimized high level model feeds into the lower level more detailed models.

The structure of the paper is the following:

Section 2 is a general introduction to the problem describing the multi-echelon supply chain and the relevant cost structure. Section 3 introduces the flexible modeling tool to build complex multi-echelon supply chain models using simple, easy to understand modules. Section 4 represents the main results through a case study, while section 5 concludes our work. The reader can follow the solution of a problem and we demonstrate the use of sensitivity analysis as a decision support tool. Finally we talk about the integration of this methodology to a complex decision support system which is part of the company Management Information System.

The proposed method

Our intent was to create a Monte-Carlo simulator which uses probability distributions based on material usage data posted in the logistic module of an ERP system. This new methodology has two main objectives.

SIMWARE can be used as a verification tool to analyze and evaluate inventory control strategies pinpoint problems due to MRP parameter determinations. The simulation of “actual” inventory controlling strategies provides the most important KPI-s of these strategies. On the other hand we can use the simulator as part of optimization and determine the optimal values of the key inventory control parameters.

SIMWARE provides a framework to analyze the cost structure and optimize inventory control parameters based on cost objectives. We are in the process to finalize the costing model therefore we used a simple cost function at this point. We have minimized the inventory holding cost by changing the parameters of our operational space while keeping the service level at the required value.

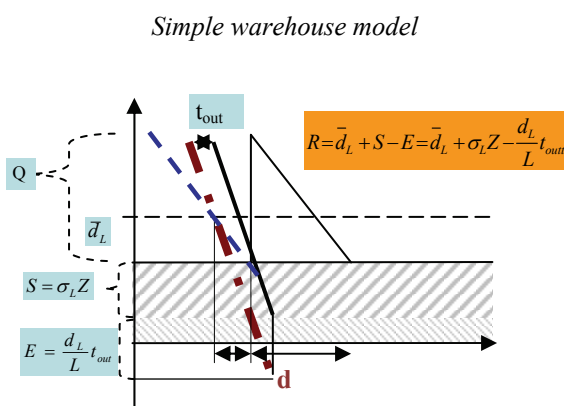


Figure 1: The classic model of inventory control.

In Fig. 1, Q is the theoretical demand over cycle time T and this is the *Order Quantity*; R is the *Reorder point*, which is the maximum demand can be satisfied during the replenishment lead time (L). S is the *Safety stock*; this is needed if the demand is higher than the expected (line d). We have a special type of Safety stock in our example. Utility companies use this stock to cover demand due to system failures.

Now we give a summary of the most important parameters of inventory control and their relevance and connection to our model.

Lead time L is the time between the Purchase order and the goods receipt. \bar{d}_L denotes the average demand during the replenishment lead time. $\bar{d}_L = \bar{d} \cdot L$, where \bar{d} is the daily average demand. Using the same logic d_L is a special case; it yields consumption if the service level is 100%. We will use d_L to denote the consumption during the manuscript. A *Cycle time* (T) is the time between two purchase orders. The *Order Quantity* is Q where $Q = \bar{d} \cdot T$. This is the ordered quantity in a purchase order.

Q is equal to the *Expected demand* and the *Maximum stock level*. *Maximum stock level* is the stock level necessary to cover the *Expected demand* in period T ; therefore it has to be the quantity we order.

Reorder point is the stock level when the next purchase order has to be issued. It is used for materials where the inventory control is based on actual stock levels.

In an ideal case R equals to total of safety stock and average demand over lead time: ($R = \bar{d}_L + S$).

S is the *Safety stock* which is to cover the stochastic demand changes and for a given *Service Level* this is the maximum demand can be satisfied over the Lead time.

Assuming constant demand pattern over the cycle time, Average Stock (K) can be calculated as follows.

$$K = \frac{Q}{2} + S \quad (1)$$

It is calculated as a weighted average of stock levels over the cycle time.

Service Level (SL) is the ratio of the satisfied and the total demand (in general this is the mean of a probability distribution), or in other words it is the difference between the 100% and the ration of unsatisfied demand:

$$SL = 100 - 100 \frac{(d_L - R)}{Q} \quad (2)$$

We assume that all demand is satisfied from stock until stock exists. When we reach stock level R the demand over the lead time (d_L) will be satisfied up to R . Consequently if $d_L > R$, we are getting a stock out situation and there will be unsatisfied demand therefore the service level will be lower than 100%. d_L is not known and it is a random variable. The probability of a certain demand level is $P(d_L)$. Based on this, the service level is formed as shown in the next equation.

$$SL = 100 - 100 \frac{\int_{d_i}^{d_{\max}} P(d_L)(d_L - R)d_L}{Q}, \quad (3)$$

where d_L is continuous random variable, and d_{\max} is the maximum demand over Lead time.

Stochastic model

Based on our experience in analyzing actual supply chain systems we discovered that the probability functions of material flow and demand are different from the theoretical functions.

As *Fig. 3* shows the distribution function of an actual material consumption is significantly different from the theoretical one (*Fig. 2*). This makes us believe that there is a difference between the theoretical (calculated) and the actual inventory movements, therefore it makes sense using an approach based on ‘‘actual’’ distribution function. We will demonstrate the results of the simulation using this approach.

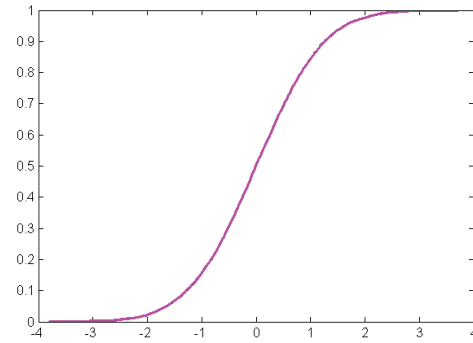


Figure 2: Theoretical Cumulative Distribution function

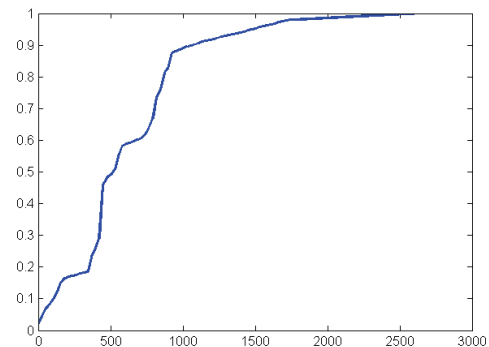


Figure 3: Actual Cumulative Distribution function for a raw material based on its consumption data

Inventory movements can be modeled much better using stochastic differential equations than modeling based on the theoretical assumption that movements are following normal distribution.

We propose the following model:

$$x_{L_{i+1}} = x_{L_i} - W_i + u(x, rop, t_u), \quad (4)$$

where x_i is stock level on the i^{th} week, W_i is a stochastic process to represent consumption. This stochastic process is based on the empirical cumulative distribution function we described in the previous section. u is the quantity of material received on week i , based on purchase orders. Purchase orders are calculated based on the actual inventory level (x), and reorder point (rop), and the replenishment lead-time (t_u).

Optimization of the system

Optimization process is changing the reorder point while keeping the service level at the required value. As a result we find the optimal reorder point for the given service level. This determines an optimal inventory level to minimize our current cost objective function.

In the first phase we created the simulator using the Sequential Quadratic Programming (SQP) functionality of MATLAB’s Optimization Toolbox.

During the optimization we calculate the minimum value of a restricted non-linear multi-variable function. In our case we use the reorder point as optimization variable (parameter), we are seeking for the minimum of the average days of inventory. Our restriction is the required value of the service level.

In the basic case we use one set random consumption data for the optimization therefore the optimum is related to this dataset.

We applied the Monte-Carlo process to overcome this problem. This is a robust methodology which generates empirical distribution functions of consumption. We use these data sets as input for the simulation model to simulate the stock movements. We use many random paths like this in every optimization step. Based on the proven convergence of the Monte-Carlo process the calculated stock movements, stock turnover and service level are good estimations of the actual process at a given reorder point.

In the current example the optimization process executes 100 simulation runs for each parameter set and uses the average of the results as objective function or constraint. We use the reorder point as optimization variable (parameter), we are seeking for the minimum of the average days of inventory. Our restriction is the required value of the service level. We demonstrate the results in the next section.

Results

In this chapter we show a complete process for the optimization of a 2-echelon supply chain, as well as we demonstrate how to solve a more complex problem by the help of our proposed SIMWARE program.

Analysis of a 2-level system

In this demonstrated example we used the simulator to analyze a system with two connected warehouses. The simulator is capable to optimize the two warehouses in the same time and calculate the optimum for the supply chain as a whole.

The following diagram shows the supply chain, i.e. the structure of the analyzed 2-level system.

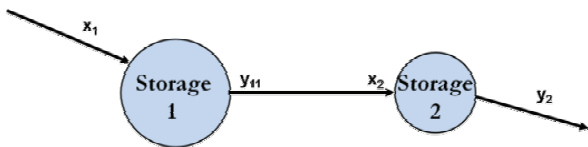


Figure 4: The analyzed 2-level system

Where the objective function is:

$$f(z) = \text{mean}(h_1) + 1.3 * \text{mean}(h_2), \tag{5}$$

i.e. the holding cost in the second Warehouse is 30% higher than in the first Warehouse.

In Fig. 5, the values of the objective function (i.e. cost) is presented as a function of the reorder point of the two Warehouses. Fig. 6 shows the service level of Warehouse 1 in the 2-level system. The constraint for the service levels is 95% in this case.

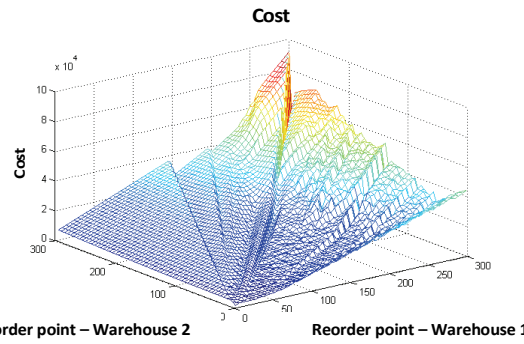


Figure 5: The values of the objective function for the 2-level system

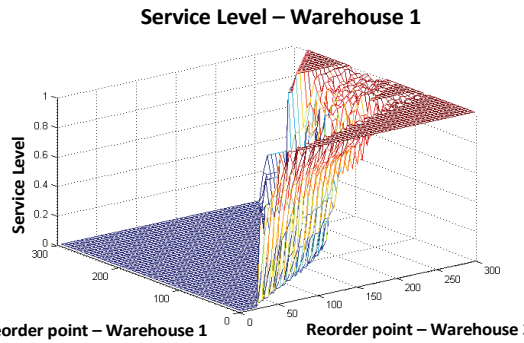


Figure 6: The values of service level for Warehouse 1.

The figure above shows only the service level for the first warehouse, but in the optimization problem both service level has to take into consideration as well as the constraints. In the next picture, we investigate all of these.

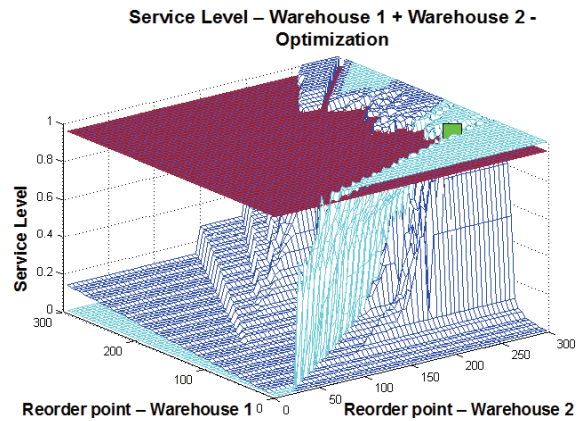


Figure 7: The result of the optimization.

Fig. 7 shows the result of the optimization using the SQP method. The optimal solution is highlighted with the green square. It satisfies the 95% constraints and ensures the minimal holding cost in the warehouses. The cost function can be seen in Fig. 5.

The result of the simulation run is presented in the next figure. The diagram shows weekly inventory levels in the Warehouses at the optimal case.

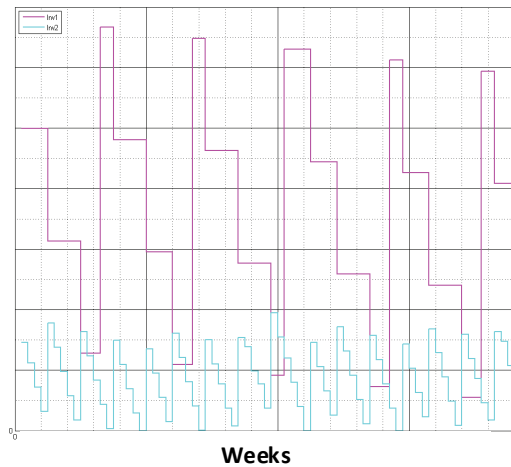


Figure 8: Inventory levels in the 2-level system.

In the rest of this section, we present a more complex, 3-level inventory model for illustration. This model can be seen in the next figure, which contains 9 warehouses in a 3-echelon structure.

In the SIMWARE program, users have to define the structure of the problem, i.e. the connection between the warehouses. The parameters of each warehouse has to be defined also, e. g. the lead time or the average demand for a product as well as the cost values, like the holding cost.

Fig. 10 represents the results of the simulation for a 3-level inventory model, where each inventory level is depicted by lines with different style.

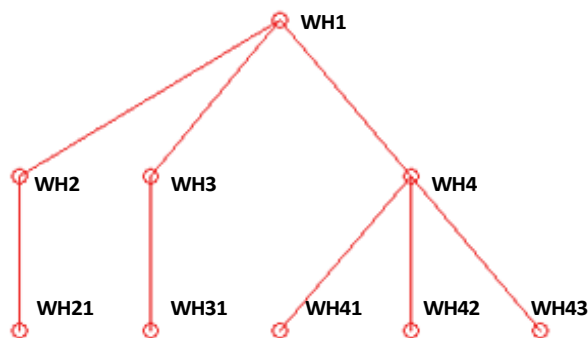


Figure 9: The analyzed 3-level system.

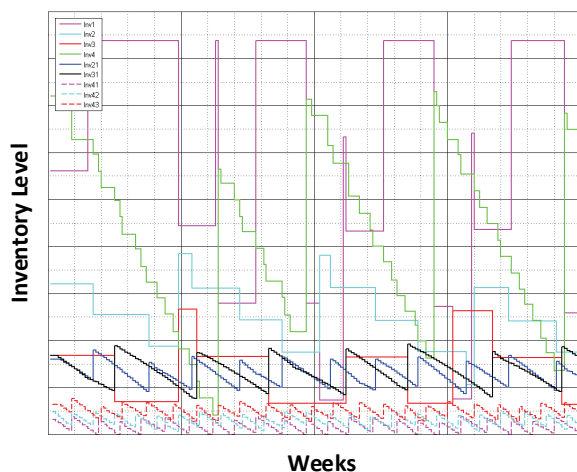


Figure 10: The inventory levels in the 3 level multi-echelon system.

Note that we have used separate distribution functions for the central warehouse and the other regional warehouses to simulate the consumption. We constructed these distribution functions based on actual data. The central warehouse consumption figures are calculated as a total of the downstream warehouse consumptions.

Using different distribution functions in a multi-echelon supply chain, or using different consumption values for each warehouse is not a trivial, but necessary task if we want to construct a more realistic model or simulator. The SIMWARE program offers an easy-to-use interface to build even complex supply chains, and propose a novel component based structure. Using this program, users can easily optimize the supply chain, for example by an SQP method, but the proposed methodology gives a chance to use more effective optimization algorithms.

Conclusions

A framework for modeling a multi-echelon supply chain has been defined. The MM and WM modules of SAP system provide data for the model for the whole supply chain. In our project we simulated a central purchasing scenario. In this supply chain the central warehouse supplies the other companies in the group. Our methodology can be used to calculate stock turnover ratio and safety stock based on the agreed service level of the warehouses. Our methodology can handle the stochastic behavior of the replenishment lead time. We used Monte-Carlo based simulation to evaluate the results and also were able to evaluate the MRP parameters used in the ERP system. Extended the model to analyze and optimize the inventory levels at the other companies in the group as well. The MATLAB simulation model can be used to determine the optimal parameters for a required service level. The SIMWARE model is available at the following website: www.folyamatmernok.hu

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