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ANALYTICAL PREDICTION OF CUTTING TOOL WEAR BY ANALYSIS OF WEAR MODELS

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The management of cutting tool wear and tool life is a long-standing topic in machining research and practice. Although a wide range of analytical and data-driven wear models has been proposed, their direct applicability in defining a robust tool life criterion and for practical tool life prediction remains limited. In this paper, we analyze several alternative functional forms for the flank wear–time (VB–t) relationship with the specific aim of defining an analytically tractable tool life criterion based on the minimum wear intensity and the inflection point of the wear curve. Linear, power, exponential and third-degree polynomial models are compared in terms of goodness of fit, monotonicity and the possibility of deriving closed-form expressions for the inflection point and the corresponding lifetime. Based on these criteria, monotonically increasing exponential and cubic polynomial models are identified as the most promising candidates. Their applicability is illustrated using a representative turning test, using a single measured wear curve as a case study rather than full statistical validation. The analysis shows that once the model parameters are identified for a given cutting system, the proposed framework can provide a transparent, analytically defined lifetime criterion and can support prediction of the remaining tool life. The work is therefore intended as a methodological contribution and as a starting point for future, more comprehensive experimental validation and for integration into digital tool condition monitoring systems.

Keywords: tool wear, tool life, wear marks, lifetime criterion, lifetime prediction, lifetime monitoring

1. Introduction

In recent years, digitalization, high-speed data (5G), especially advanced sensor technology [1], artificial intelligence (AI) [2] and, related to these, digital twin [3] have led a number of researchers to work on the wear process of cutting tools, tool life, its predictability and automatic tool monitoring [4],[5] or other international programs in this field, in view of the fact that Industry 4.0 is the fourth digital industrial revolution, in which information technology (IT) and automation are becoming increasingly intertwined, leading to a fundamental change in manufacturing methods, where sensors, machines, workpieces (parts) and IT systems are linked along the manufacturing process [6].

Numerous approaches have been proposed for tool condition monitoring and tool wear prediction in turning and milling, ranging from physics-based models to machine-learning-based solutions. Nevertheless, in many practical applications the tool life is still determined indirectly, using relatively simple analytical wear models and empirically selected wear limits. The aim is always to produce accurately, cost-effectively and in an environmentally friendly way, and this is inextricably linked to making the most of the knowledge and tools available [7]. This is one of the reasons why further analysis of the wear mechanism of cutting tool edges, the causes and the process of wear are necessary. For adaptive optimization of cutting parameters, multisource machining data can be used (e.g. cutting force [8],[9] or vibration [10],[11]), which help to set up a tool wear prediction model. In several studies [12]-[15] the tool wear prediction models were investigated using multisource matching data. In addition to the multisource machining data, wear is greatly influenced by the right choice workpieces and cutting tool material [16]. Marousi et al. [17] came to the conclusion that even the grain structure of the material is a highly influential factor in wear, also stated that fractal analysis of signals can be used to monitor the tool wear. Then there are studies in which the effect of clamping and possible displacement of the workpiece and the tool on wear is also examined; with the knowledge, data can be used to filter out most of negative factors [18].

With the knowledge of all kinds of data (e.g. from tool makers), the expected tool life can be predicted approximately, but unfortunately in practice the precise prediction of tool wear using traditional machine learning methods is highly challenging due to variable cutting conditions and the limited availability of tool wear data. To address these dual challenges, Biyao Qiang et al. [19]

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propose a physics-informed transfer learning (PITL) framework for predicting tool wear under varying working conditions. Of course, there are other models that determine the life expectancy based on different methods [20]. Li et al. [21] introduced a data-driven tool wear monitoring approach using radar images. The method integrates radar image features and utilizes two models: a decision tree based on AdaBoost and a stacked bidirectional long short-term memory (BiLSTM) algorithm. Together, these models enable both the recognition of tool wear states and the prediction of tool wear progression.

The problem with most of these models is that they are not capable of online monitoring, with their help, tool life can only be determined afterwards. The reason for this is that the models that work with the geometric dimensions [22] of the wear are much more accurate [23],[24], this measurement could only be performed by pausing (offline) the cutting process. Models that work with physical data obtained during cutting [25] must be much more complex, because that is the only way to provide valid results. Pengfei Ding et al. [26] propose a time-varying cutting force modelling method that accounts for tool wear, deformation, workpiece size change, elastic recovery and chip separation. There are also models that can be used to pre-determine (with reservations, of course) the tool life, without any cutting. Nanyuan Zhang et al. [27] present an efficient approach of tool wear simulation using the numerical SPH-FEM method.

From the literature, it can be concluded that usable wear analysis and tool life prediction can only be obtained from operational data (wear, temperature, performance or power). The most effective of these are offline wear measurement [28]-[30] and power measurement [31]. Wear measurement can be performed on tools that are currently outside the grip, in the tool magazine or in the turret head. The cutting power can be continuously monitored and compared to that produced by the original wear-free tools (MAZAK application) [32].

Against this background, there is still a need for transparent, analytically tractable wear models that can be calibrated for a given cutting system and then used to define and predict tool life in a consistent way. The present work therefore focuses on the analytical description of the flank wear–time relationship $VB(t)$ and on the definition of a tool life criterion that is consistent with the physical shape of the wear curve. The specific objectives are:

- to compare several candidate functional forms (linear, power, exponential and cubic polynomial) in terms of goodness of fit, monotonicity and analytical tractability,
- to derive closed-form expressions for the inflection point of $VB(t)$ and for the minimum wear intensity, and to relate these to a rational lifetime criterion and
- to illustrate the practical use of the most promising models on a representative turning test, based on a single measured wear curve.

The study is intended as a methodological exploratory contribution. It does not aim at a full experimental or preliminary analytical study or at developing a complete real-time monitoring system, but at providing a mathematically transparent framework that can be combined with different measurements and monitoring strategies in future work.

1.1. Causes of wear

Based on a wide range of research over the past century, researchers still agree that wear on cutting tools is caused by several physical, mechanical and chemical effects, including [33]:

- abrasive wear,
- adhesion wear,
- diffusion wear,
- oxidation wear,
- chipping wear and
- wear resulting from plastic deformation.

The main wear mechanisms occurring during machining include abrasive wear, adhesive wear, oxidation wear, and diffusion wear. These mechanisms primarily affect the rake face and flank face of the cutting tool, thereby influencing tool life, surface integrity, and overall machining performance [34]. The proportion of these effects depends on the specific technological conditions, but it can be concluded that the thermal effect of the machining process plays a decisive role in their development and significance [35].

Koren [36] and Roumesy [37] also confirm that if a tool is operated under the technological conditions recommended by the manufacturers for the workpiece material, without edge deformation and rapid edge burnout (oxidation), then abrasion and diffusion effects are the most important for the development of wear. Abrasive wear typically increases rapidly at the beginning of machining and then tends to stabilise, whereas diffusion wear becomes increasingly significant with longer cutting times. The combined effect of these two mechanisms determines the characteristic overall wear behaviour of the cutting tool as a function of machining time.

1.2. The process of wear

The edges of cutting tools wear and tear during machining. The extent of wear is usually described by the average wear (VB) measured on the major flank (*Figure 1*), according to ISO 3685, the generally allowable wear rate for high-speed steel and carbide $VB_{allowed}$ = 0.3 mm, and for ceramics $VB_{allowed}$ = 0.2 mm [38]. Reaching this value represents the theoretical tool edge failure time, the tool life (T). The average back wear is the average wear over the edge length $b - (b/4 + r_ε)$, as shown in the figure. b is the original edge length of the tool, which is equal to the theoretical chip width. The maximum back wear VB_{max} is most often measured near the tool tip or at the end of the tool length in the grip, the allowable magnitude of which is generally recommended to be twice the value of $VB_{allowed}$.

When machining tough materials, cratering wear develops on the front surface, the extent of which can be described by the crater depth (*KT*), but it is difficult to measure, tool wear is most commonly characterized by the *VB–t* diagram, i.e. the wear curve. This wear curve can generally be divided into three stages. In stage I, wear increases relatively rapidly and is dominated mainly by abrasion. In stage II, wear develops approximately linearly, while diffusion becomes increasingly significant due to the temperature rise associated with the wear process. In stage III, wear accelerates progressively, leading ultimately to complete tool failure [39].

Table 1 summarizes the characteristics of the three stages of the wear curve and the reasons for the increase in wear. Phase II is represented by the author as a simple straight line, although it is only an approximation, since at some point the degressive phase changes into a progressive one, which means that there must be an inflection point at the transition, the exact interpretation and definition of which are among the cornerstones of our research [40].

From a practical point of view, the interpretation and definition of the tool life criterion are essential when analysing the wear curve. In general, the allowable wear value, in this case the allowable flank wear VB_{allowed}, has traditionally been determined based on machining tests and manufacturing experience. However, this approach may lead either to underutilization or over-utilization of the cutting edge. For different cutting speeds ($v_{c1}<v_{c2}<v_{c3}$), the tool lives determined at the intersection of the wear curves with VB_{allowed} (T_1, T_2, T_3) do not always coincide with the lifetimes corresponding to the maximum permissible wear. In the case of the wear curve for v_{c1}, the tool life meets the predefined criterion with nearly maximum utilization, whereas for v_{c2} the available tool life can be utilized more effectively, and for v_{c3} there may still be a reserve in tool life before the maximum wear limit is reached. By contrast, if the cutting edge is used beyond the permissible limit, over-utilization may occur, which can result in tool breakage or in unacceptable deterioration of the surface roughness and dimensional accuracy of the machined workpiece.

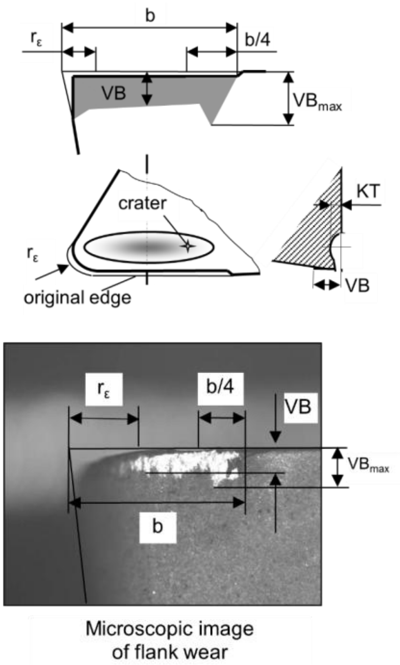

Figure 1: Interpretation of wear types

Applying VB_{allowed} according to the ISO recommendation on the v_{c1} curve, we entered the stage of dangerous progressive wear, i.e. ***T1***<T1. On the v_{c3} curve, we did not use the performance of the tool because we stopped before the start of the progressive phase, so ***T3***> T3. We stopped on the v_{c2} curve just at the beginning of the progressive phase, we also used the edge of the tool with VB_{allowed}, i.e. ***T2***=T2.

Table 1: Analysis of wear curve sections [40]

Characteristics	I. Degressive wear	II. Linear wear	III. Progressive wear
Visual Clues	• Repetitive radius of curvature of chip with smooth reflective underside • Part surface finish smooth • Slight flank wear	• Chips begin to show changing curvature and duller underside • Surface finish on part poorer • Collar begins to form • More flanks wear	• Chips irregular and broken with uneven serrations and dull underside • Dull surface finish • Large collar • Heavy flank wear
Aural Cues	• “Clean hiss” of serrations and “constant trickle” of regular chip sections • No unusual “spikes”	• Less regular chip sections interrupt even “trickle” • Some “spikes” heard	• Chips obviously uneven • Spikes intense with squealing possible
Tactile Cue	• Part surface feels smooth	• Some “ears” on part surface	• Rough “ears” on part
Machinist’s Thoughts	• Tool basically “new” and working smoothly	• Expected deterioration occurring	• Must stop before damage occurs

T1, T2, T3 - obtained when applying VB_{allowed}; ***T1, T2, T3*** – lifetime according to the maximum allowable wear; v_{c1}, v_{c2}, v_{c3} - cutting speeds for wear curves; *t* - cutting time; *VB*– flank wear

2. Tool life criterion, the possibility of predicting life

The maximum allowable wear rate is defined by the minimum wear intensity:

$$I = \frac{VB(t)}{t} \rightarrow minimum \qquad (1).$$

The wear intensity is the ratio of the instantaneous wear to the time elapsed since the chipping, after which the wear curve enters the third, intense over-wear phase. There is an inflection between the initial and maximum allowable wear, where the degressive phase enters the progressive range, i.e.

$$v_w = VB(t)' = \frac{dVB}{dt} = 0 \qquad (2).$$

The wear rate (v_w) and the wear intensity (I) derived from the wear curve are plotted with the inflection (t_{inf}) and the lifetime (T) in *Figure 2*.

The minimum values can be determined by an extreme value calculation after derivation of the wear function fitted to the measured points. As described above, the following requirements can be made of the fitted functions:

1. as close to reality as possible, with a good fit to the measured data,
2. provide a simple way to determine the lifetime (***T***), i.e. to calculate the minimum of the wear intensity (I) in a closed form,
3. compute the inflection point t_{inf} of the function *VB*-t in closed form,
4. there should be a clear, numerical relationship between t_{inf} and ***T*** to predict the expected occurrence of the end of life and
5. it should preferably be monotonically increasing, as wear increases monotonically during machining.

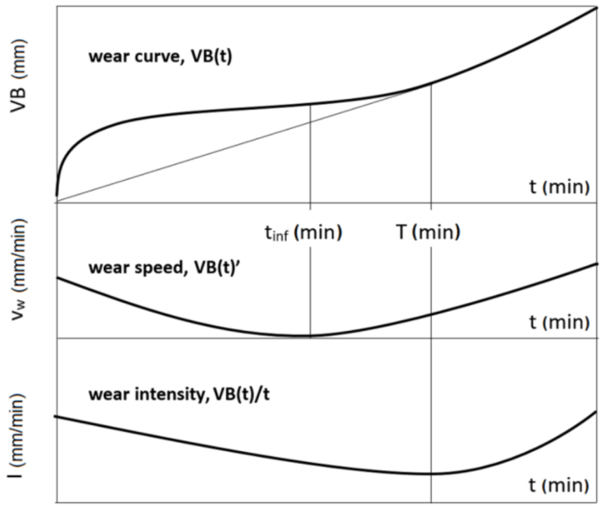

Figure 2: The wear pit, wear rate and wear intensity

2.1. Functions fitted to the measured values of the wear curve VB (t)

In this research, we first examined the goodness-of-fit of the traditionally used linear and power functions fitting *VB* points to real-world *VB* using Microsoft Excel®. As an example, *Figure 3* illustrates that their correlation coefficients are below the R>0.900 value still considered acceptable in engineering practice.

Based on the measurements and the literature [11], we tried exponential functions followed by third degree polynomials on several *VB* data sets. Some of these relationships can be found in the literature. The functions are the followings (I.-V.):

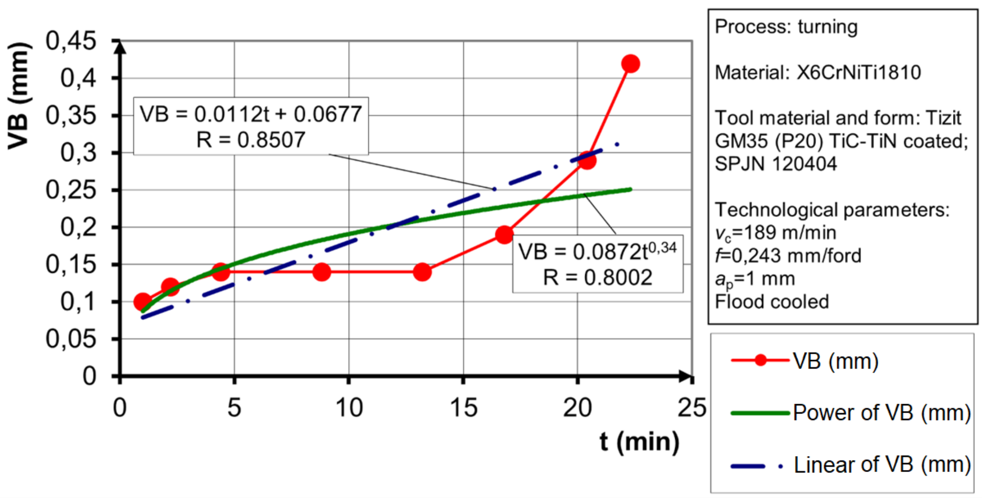

Figure 3: Linear and power function fitting

***I. Sum of two exponential functions* (VB. exp. exp) [11]**

The function

$$VB = A(1 - e^{-\alpha t}) + B\left(e^{\beta t} - 1\right) \quad (3),$$

where A, B, α, β are the constants of the equation to be determined during function fitting.

The inflection point

$$t_{inf} = \frac{1}{\alpha+\beta} \cdot ln \frac{A \cdot \alpha}{B \cdot \beta} \quad (4)$$

The tool lifetime

To calculate the minimum wear intensity, the following transcendental equation was obtained:

$$-A + B + Ae^{-\alpha t}(\alpha t + 1) + Be^{\beta t}(\beta t - 1) = 0 \quad (5),$$

which cannot be solved analytically; therefore the lifetime $\boldsymbol{T}$ cannot be calculated in closed form, and there is no numerical relationship between t_{inf} and $\boldsymbol{T}$.

***II. Exponential, non-monotonically increasing* (VB.exp. n.mon) [14]**

The function

$$VB = t \cdot e^{A+Bt+Ct^2} \quad (6),$$

where A, B, C are the constants of the equation.

The inflection point

$$t_{inf1,2} = \frac{-B \mp \sqrt{B^2 - 8C}}{4C} \quad (7)$$

If $B^2 - 8C > 0$, between the two extremes, the slope of the curve is negative, i.e. the wear decreases over time along this section, which is physically impossible. The curve is not monotonically increasing.

The tool lifetime

$$\boldsymbol{T} = -\frac{B}{2C} \quad (8)$$

***III. Exponential, monotonically increasing* (VB.exp.mon)**

The function

$$VB = t \cdot e^{A+Bt+\frac{B^2}{8t^2}} \quad (9),$$

where A and B are the constants of the equation.

The inflection point

$$t_{inf} = -\frac{2}{B} \quad (10)$$

The tool lifetime

$$\boldsymbol{T} = -\frac{4}{B} \quad (11),$$

i.e.

$$\boldsymbol{T} = 2 \cdot t_{inf} \quad (12).$$

***IV. Polynomial, non-monotonically increasing* (VB.pol. n.mon)**

The function

$$VB = A \cdot t^3 + B \cdot t^2 + C \cdot t \quad (13),$$

where A, B, C are the constants of the equation.

The inflection point

$$t_{inf} = -\frac{B}{3A} \quad (14)$$

The tool lifetime

$$\boldsymbol{T} = -\frac{B}{2A} \quad (15),$$

i.e.

$$\boldsymbol{T} = \frac{3}{2} t_{inf} \quad (16).$$

Note that the non-monotonic increasing nature of the function allows for "backtracking", a reduction in wear in the process, which cannot occur in reality.

***V. Polynomial, monotone increasing* (VB.pol.mon)**

The function

$$VB = A \cdot t^3 + 3 \cdot A \cdot C \cdot t^2 + 3 \cdot A \cdot C^2 \cdot t \quad (17),$$

where A, C are the constants of the equation.

The inflection point

$$t_{inf} = -C \quad (18)$$

The tool lifetime

$$\boldsymbol{T} = -\frac{3}{2} C \quad (19),$$

i.e.

$$\boldsymbol{T} = \frac{3}{2} t_{inf} \quad (20).$$

2.2. Degree of goodness of fitted curves, practical usefulness based on measured data

Several tens of wear curves were recorded on a universal lathe, where the workpiece was chucked and tip-supported. In the present manuscript, detailed plots are shown for one representative wear curve in order to facilitate the analytical discussion of the different model structures. The remaining curves were used to verify qualitatively that the main observations are not limited to a single experiment, but no full statistical evaluation was carried out.

The workpieces diameter was between 60-120 mm and the length was 400 mm; the material grade was austenitic corrosion resistant steel X6CrNiTi1810. The HSS-E high-speed steel, uncoated and CERATIZIT brand GM35 TiC-TiN coated P20 carbide plate were tested. The MITUTOYO Quick Vision ELF Pro measuring microscope and workshop microscope were used for wear measurement. Based on the criteria

Table 2: Evaluation of fitted functions

Sign of function		Criteria 1.	2.	3.	4.	5.
I.	**VB.exp.exp**	+	-	+	-	+
II.	**VB.exp.n.mon**	+	+	-	-	-
III	**VB.exp.mon**	+	+	+	+	+
IV	**VB.pol.n.mon**	+	+	+	+	-
V.	**VB.pol.mon**	+	+	+	+	+

described in Chapter 2, the functions discussed in Chapter 2.1 were evaluated in *Table 2* and fitted to the measured points using VisualBasic® software. It can be seen that for the GM35 carbide plate all functions satisfy criterion I., but only III and V. fully satisfy the others.

The evaluation of *Table 2* is supported by *Figure 4*. It can be seen that the non-monotonic increasing functions (**VB.exp.n.mon**, **VB.pol.n.mon**) can have several inflection points, which precludes the possibility of a concrete prediction of the lifetime, unlike the monotonic increasing functions (**VB.exp.mon**, **VB.pol.mon**), where the end of the lifetime can be precisely determined by knowing the single inflection.

In *Table 3*, the coefficients of the function curves shown in *Figure 4*, the corresponding inflection points, edge holdings and correlation indices are given, which can be used to write the concrete equations. The correlation indices close to R=1.00 stand out, in contrast to the values of R=0.80 and 0.85 in *Figure 3*, noting that similar values were observed for wear curves recorded under other technological conditions. To demonstrate the results, we chose the technological set-up shown in *Figure 3* because tooling companies recommend a tool life of $\boldsymbol{T}$=10-20 min for carbide flap tools, considering economic and productivity aspects, and the tool life falls within this range.

It should be emphasised that the very high correlation coefficients reported in *Table 3* quantify only the goodness of fit on the same experimental data that were used to identify the model parameters. They do not represent predictive accuracy in the sense of forecasting wear on independent experiments. If the turning tests were repeated several times under identical cutting conditions, some scatter of the measured wear curves and tool life would be expected, and the errors of the fitted

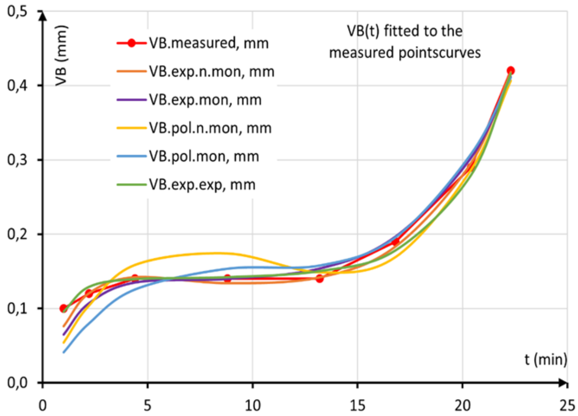

Figure 4: Exponential functions and polynomials of degree three fitted to the measured values in *Figure 3*

models with respect to individual repetitions would inevitably increase. Quantifying such predictive performance would require a dedicated experimental campaign with repeated tests and an appropriate statistical analysis, which is beyond the scope of this preliminary study.

2.3. The possibility of lifetime forecasting

For most of the recorded wear curves, we found that, by connecting the measured points, the inflection falls within two-thirds of the distance from the origin to the start of the real phase III. This suggests that the real conditions are best approximated by the **VB.pol.mon** functions, as confirmed in *Table 4*.

Examining *Table 4* and *Figure 5*, fitting polynomials to the measured wear intensity and measured wear rate plots, we obtain a confirmation of the relationship $\boldsymbol{T}$≈1.5t_{inf} according to *Equation 20*, which agrees numerically well with the calculated data in the **VB.pol.mon** column of *Table 3*. This agreement suggests that, in principle, dense wear measurements combined with rapid numerical evaluation could allow the end of tool life to be anticipated before the conventional *VB* criterion is reached, typically by about 25–30% of the service time in the examples studied. However, this should currently be regarded as a conceptual demonstration based on limited experimental data. A

Table 3: Numerical data for the fitted functions in *Figure 4*

Title	Signal	**VB.exp. n.mon**	**VB.exp. mon**	**VB.pol. n.mon**	**VB.pol. mon**	**VB.exp. exp**
Coefficient	A	-2.280	-2.480	0.0002	0.000145	0.140
Coefficient	B	-0.310	-0.270	-0.00633		$9.00\text{x}10^{-5}$
Coefficient	C	$1.04\text{x}10^{-2}$		$6.00\text{x}10^{-2}$	-1.02x10	
Coefficient	α					1.150
Coefficient	β					0.360
VB(t) inflection point	t_{inf}, (min)	----	7.460	10.560	10.200	5.640
Minimum wear intensity	$\boldsymbol{T}$, (min)	14.810	14.930	15.830	15.300	----
VB(t) correlation index	R	0.9971	0.9908	0.9740	0.9752	0.9976

Table 4: Wear intensity ($I_{measured}$) and wear rate ($v_{w\ measured}$) values for the measured $VB_{measured}$ data in *Figure 4*

t (min)	$VB_{measured}$ (mm)	$I_{measured}$ (mm/min) $VB_{measured}/t$	Δt (min)	ΔVB (mm)	$v_{w\ measured}$ (mm/min) $\Delta VB/\Delta t$
0.1	0.02	0.200			
1.0	0.10	0.100	0.9	0.08	0.089
2.2	0.12	0.055	1.2	0.02	0.017
4.4	0.14	0.032	2.2	0.02	0.009
8.8	0.14	0.016	4.4	0.00	0.000
13.2	0.15	0.011	4.4	0.01	0.002
16.8	0.19	0.011	3.6	0.04	0.011
20.4	0.29	0.014	3.6	0.1	0.028
22.3	0.42	0.019	1.9	0.13	0.068

statistically robust assessment of prediction accuracy would require repeated measurements under identical cutting conditions and an evaluation of the scatter of the $\boldsymbol{T}/t_{inf}$ relation, which is planned for future work.

From a practical point of view, implementing real-time tool life prediction with the proposed approach would require several additional steps. First, the chosen analytical model structure (e.g. the monotonically increasing third-order polynomial **VB.pol.mon**) would need to be calibrated offline for a given combination of tool, workpiece and cutting parameters, using several experimentally measured wear curves to capture the variability of tool life. During production, wear could then be estimated either intermittently from direct measurements of *VB* or indirectly from process signals, and the analytical relation between t_{inf} and $\boldsymbol{T}$ would provide an estimate of the remaining tool life. The present paper addresses only the analytical foundations of this approach and an illustration based on limited experimental data; a full implementation and validation of a real-time monitoring system is left for future work.

Wear measurement during milling – using a shape recognition system – can even be carried out online, when the measured blade is briefly out of grip. Fast and accurate evaluation is ensured by modern IT tools and AI systems [2],[3]. In turning, measurements are usually only possible in an off-line mode, e.g. during tool change, which is a more complex and slower process.

As already mentioned in the introduction, besides the lifetime prediction, it is of course necessary to monitor the machining process in the other direction, detecting accidental collisions and tool breakage, e.g. by force measurement. If the measured torque or force component exceeds an allowable limit, the process is stopped immediately. Such a system is offered by tool manufacturer CERATIZIT, for example. This ToolScope tool monitoring system detects collisions, breakage and also over-abrasion that has already occurred (without prediction!) during drilling and milling; it can automatically stop the process [40],[42].

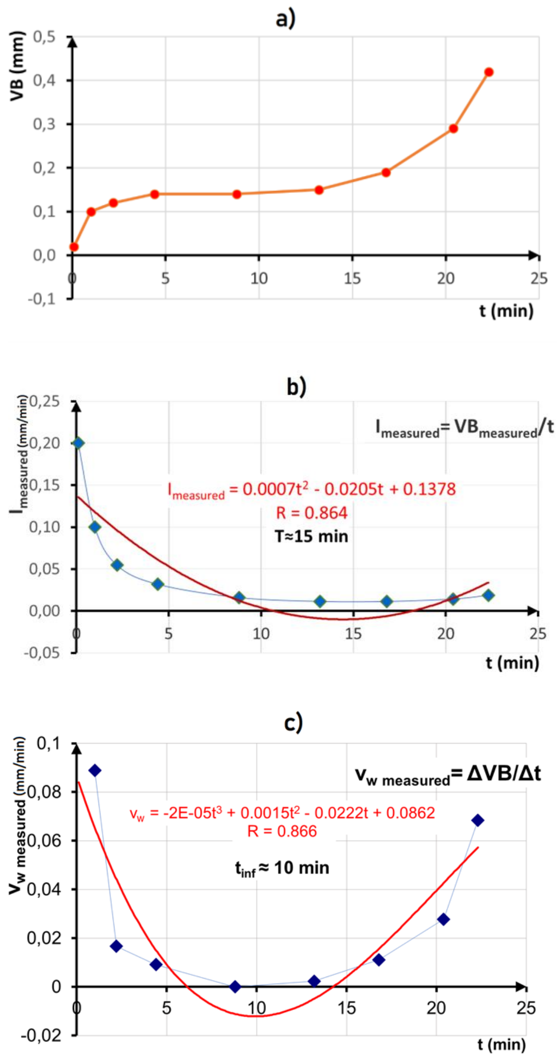

Figure 5: Plots of the a) measured points ($VB_{measured}$) of wear curve, b) measured wear intensity (Imeasured) and c) the wear speed ($v_{w\ measured}$) based on the fitted polynomials

Once the model parameters have been identified for a given cutting system, the *VB(t)* function and its inflection point can be used to define a consistent lifetime criterion and to estimate the remaining tool life from the current wear state. In this sense, the proposed analytical framework can support real-time or near-real-time prediction, provided that it is combined with an appropriate wear measurement or state estimation method. The present paper, however, is limited to the analytical aspects and the demonstration on a single wear curve; a systematic evaluation of prediction accuracy under repeated experiments remains a subject of future work.

3. Summary

In this publication, the authors have analyzed the causes and processes of cutting tool wear based on literature data and their own experimental experience and combined this with an analytical investigation of several wear model structures. A lifetime criterion based on the minimum wear intensity and the inflection point of the *VB–t* curve has been formulated and expressed in closed form for selected exponential and polynomial models. The analytical relations between the inflection time $\boldsymbol{t}_{\text{inf}}$ and the tool life $\boldsymbol{T}$ have been illustrated on experimentally measured wear curves from turning tests, showing that a relation of approximately $\boldsymbol{T} \approx 1.5\, \boldsymbol{t}_{\text{inf}}$ holds for the analyzed datasets. In this way, new wear model structures have been set up to support the monitoring of tool life and the cutting process, and to provide a potential basis for improving the accuracy and reliability of tool life prediction. With the application of modern IT tools and artificial intelligence (AI), the proposed analytical framework could contribute to the increasingly widespread field of digital twin technology, especially in the improvement of monitoring systems, for example in analyzing wear of machine tool components and predicting failure.

The present study has several limitations. The analytical comparison of the candidate wear models and the derivation of the lifetime criterion are illustrated on a limited number of measured wear curves, in particular on a single representative curve without repeated experiments under identical conditions. As a consequence, the reported goodness-of-fit and the apparent prediction accuracy reflect these specific datasets and cannot be interpreted as a statistical validation. It is expected that, if the wear tests were repeated many times, the prediction accuracy would decrease due to experimental variability and measurement noise. Furthermore, the study does not include any direct evaluation of online or real-time monitoring strategies; it is restricted to the analytical aspects of VB(t) modelling. A comprehensive assessment of prediction accuracy and robustness, based on repeated measurements and different cutting conditions, as well as the integration of the calibrated models with online sensing and data processing in tool condition monitoring and digital-twin systems, is left for future work.

REFERENCES

[1] Aghdam, B.H.; Vahdati, M.; Sadeghi, M.H.: Vibration-based estimation of tool major flank wear in a turning process using ARMA models, *Int. J. Adv. Manuf. Technol.*, 2015, **76**(9-12), 1631–1642, DOI: 10.1007/s00170-014-6296-3

[2] Colantonio, L.; Equeter, L.; Dehombreux, P.; Ducobu, F.: A systematic literature review of cutting tool wear monitoring in turning by using Artificial Intelligence techniques, *Machines,* 2021, **9**(12), 351, DOI: 10.3390/machines9120351

[3] Qiao, Q.; Wang, J.; Ye, L.; Gao, R.X.: Digital twin for machining tool condition prediction, *Procedia CIRP*, 2019, **81**, 1388–1393, DOI: 10.1016/j.procir.2019.04.049

[4] Yoo, Y.; Yang, G.; Park, K.; Hyun, Y.; Jeong, S.: Extendable machine tool wear monitoring process using image segmentation based deep learning model and automatic detection of depth of cut line, *Eng. Appl. Artif. Intell.,* 2024, **135**, 108570, DOI: 10.1016/j.engappai.2024.108570

[5] Domínguez-Monferrer, C.; Fernández-Pérez, J.; De Santos, R.; Miguélez, M.H.; Cantero, J.L.: Machine learning approach in non-intrusive monitoring of tool wear evolution in massive CFRP automatic drilling processes in the aircraft industry, *J. Manuf. Syst.* 2022, **65**, 622–639, DOI: 10.1016/j.jmsy.2022.10.018

[6] Munirathinam, S.: Chapter six - Industry 4.0: Industrial Internet of Things (IIOT), in: Advances in Computers, (Raj, P.; Evangeline, P. (Eds.) (Academic Press Inc.) 2020, pp. 129–164, DOI: 10.1016/bs.adcom.2019.10.010

[7] Li, C.; Zhao, X.; Cao, H.; Li, L.; Chen, X.: A data and knowledge-driven cutting parameter adaptive optimization method considering dynamic tool wear, *Robot. Comput.–Integr. Manuf.*, 2023, **81**, 102491, DOI: 10.1016/j.rcim.2022.102491

[8] Liu, T.; Song, J.; Zhang, K.; Liu, Q.; Chen, F.: Tooth-wise monitoring of the asymmetrical tool wear in micro-milling based on the chip thickness reconstruction and cutting force signal, *Mech. Syst. Signal Process*, 2024, **208**, 111004, DOI: 10.1016/j.ymssp.2023.111004

[9] Pittalà, G.M.; Rizzuti, S.: An investigation of the effect of tool wear on cutting force coefficients for solid end mills, *Procedia CIRP*, 2023, **117**, 444–449, DOI: 10.1016/j.procir.2023.03.075

[10] Gao, G.; Xia, Z.; Su, T.; Xiang, D.; Zhao, B.: Cutting force model of longitudinal-torsional ultrasonic-assisted milling Ti-6Al-4V based on tool flank wear, *J. Mater. Process. Technol.*, 2021, **291**, 117042, DOI: 10.1016/j.jmatprotec.2021.117042

[11] Zhang, P.; Gao, D.; Lu, Y.; Ma, Z.; Wang, X.; Song, X.: Cutting tool wear monitoring based on a smart toolholder with embedded force and vibration sensors and an improved residual network, *Measurement*, 2022, **199,** 111520, DOI: 10.1016/j.measurement.2022.111520

[12] Cheng, W.-N.; Cheng, C.-C.; Lei, Y.-H.; Tsai, P.-C.: Feature selection for predicting tool wear of machine tools, *Int. J. Adv. Manuf. Technol*. 2020, **111**(5-6), 1483–1501, DOI: 10.1007/s00170-020-06129-5

[13] Shi, K.N.; Zhang, D.H.; Liu, N.; Wang, S.B.; Ren, J.X.; Wang, S.L.: A novel energy consumption model for milling process considering tool wear progression. *J. Clean. Prod*. 2018, **184**, 152-159, DOI: 10.1016/j.jclepro.2018.02.239

[14] Zhang, L.; Zhao, X.; Ke, Q.; Dong, W.; Zhong, Y.: Disassembly line balancing optimization method for high efficiency and low carbon emission, *Int. J. Precis. Eng. Manuf.-Green Technol.*, 2021, **8**(1), 233–247, DOI: 10.1007/s40684-019-00140-2

[15] Yu, J.: Tool condition prognostics using logistic regression with penalization and manifold regularization, *Appl. Soft Comput.*, 2018, **64**, 454-467, DOI: 10.1016/j.asoc.2017.12.042

[16] Liang, J.; Gao, H.; Li, D.; Lei, Y.; Li, S.; Guo, L.; Chen, L.; Leng, Z.; Sun, Y.; Li, C.: Study on milling tool wear morphology and mechanism during machining superalloy GH4169 with PVD-TiAlN coated carbide tool, *Tribol. Int.*, 2023, **182**, 108298, DOI: 10.1016/j.triboint.2023.108298

[17] Marousi, M.; Rimpault, X.; Turenne, S.; Balazinski, M.: Initial tool wear and process monitoring during titanium metal matrix composite machining (TiMMC), *J. Manuf. Process.*, 2023, **86**, 208–220, DOI: 10.1016/j.jmapro.2022.12.047

[18] Zhu, K.: Smart machining systems - Modelling, monitoring and informatics (Springer Cham), 2022, DOI: 10.1007/978-3-030-87878-8

[19] Qiang, B.; Shi, K.; Liu, N.; Ren, J.; Shi, Y.: Integrating physics-informed recurrent Gaussian process regression into instance transfer for predicting tool wear in milling process, *J. Manuf. Syst.*, 2023, **68**, 42–55, DOI: 10.1016/j.jmsy.2023.02.019

[20] Klocke, F.; Gorgels, C.; Herzhoff, S.: Tool load during multi-flank chip formation, *Adv. Mater. Res.*, 2011, **223,** 525–534, DOI: 10.4028/www.scientific.net/AMR.223.525

[21] Li, X.; Liu, X.; Yue, C.; Liu, S.; Zhang, B.; Li, R.; Liang, S.Y.; Wang, L.: A data-driven approach for tool wear recognition and quantitative prediction based on radar map feature fusion, *Measurement*, 2021, **185**, 110072, DOI: 10.1016/j.measurement.2021.110072

[22] Yu, W.; Ming, W.; An, Q.; Chen, M.: Wear behavior of SiAlON ceramic tool and its effects during high-speed cutting, *Ceram. Int*. 2023, **49**(16), 26694–26706, DOI: 10.1016/j.ceramint.2023.05.205

[23] Wang, P.; Bai, Q.; Cheng, K.; Zhang, Y.; Zhao, L.; Ding, H.: Investigation on an in-process chatter detection strategy for micro-milling titanium alloy thin-walled parts and its implementation perspectives, *Mech. Syst. Signal Process*., 2023, **183**, 109617, DOI: 10.1016/j.ymssp.2022.109617

[24] Chen, N.; Li, H.N.; Wu, J.; Li, Z.; Li, L.; Liu, G.; He, N.: Advances in micro milling: From tool fabrication to process outcomes, *Int. J. Mach. Tools Manuf.*, 2021, **160,** 103670, DOI: 10.1016/j.ijmachtools.2020.103670

[25] Zhou, X.; Yu, T.; Wang, G.; Guo, R.; Fu, Y.; Sun, Y.; Chen, M.: Tool wear classification based on convolutional neural network and time series images during high precision turning of copper, *Wear*, 2023, **522**, 204692, DOI: 10.1016/j.wear.2023.204692

[26] Ding, P.; Huang, X.; Zhao, C.; Liu, H.; Zhang, X.: Online monitoring model of micro-milling force incorporating tool wear prediction process, *Expert Syst. Appl.,* 2023, **223**, 119886, DOI: 10.1016/j.eswa.2023.119886

[27] Zhang, N.; Klippel, H.; Kneubühler, F.; Afrasiabi, M.; Röthlin, M.; Kuffa, M.; Bambach, M.; Wegener K.: Study on the effect of wear models in tool wear simulation using hybrid SPH-FEM method, *Procedia CIRP*, 2023, **117**, 414–419, DOI: 10.1016/j.procir.2023.03.070

[28] Čerče, L.; Pušavec, F., Kopač, J.: 3D cutting tool-wear monitoring in the process, *J. Mech. Sci. Technol*., 2015, **29**(9), 3885–3895, DOI: 10.1007/s12206-015-0834-2

[29] Čerče, L.; Pušavec, F., Kopač, J.: Novel spatial cutting tool-wear measurement system development and its evaluation, *Procedia CIRP,* 2015, **37**, 170–175, DOI: 10.1016/j.procir.2015.08.058

[30] Xiong, G.; Liu, J.; Avila, A.: Cutting Tool Wear Measurement by Using Active Contour Model Based Image Processing, in: 2011 IEEE International Conference on Mechatronics and Automation (ICMA), Beijing, China (IEEE), 2011, pp. 670–675, DOI: 10.1109/ICMA.2011.5985741

[31] Cuppini, D.; D'errico, G.; Rutelli, G.: Tool wear monitoring based on cutting power measurement, *Wear*, 1990, **139**(2), 303–311, DOI: 10.1016/0043-1648(90)90052-C

[32] Mazak Optonics Corp: What are the Mazak Intelligent Functions? https://www.mazakoptonics.com/news-events/blog/what-are-the-mazak-intelligent-functions (accessed: 08/19/2024)

[33] Astakhov, V.P.: Tribology and interface engineering series vol. 52 - Tribology of metal cutting (Elsevier Science), 2006, ISBN: 9780444528810

[34] Klocke, F.; König, W.: Fertigungsverfahren1 - Drehen, Fräsen, Bohren (8th edition) (Springer Berlin, Heidelberg), 2008, DOI: 10.1007/978-3-540-35834-3

[35] Holmberg, K.; Matthews, A.: Tribology and interface engineering vol. 56 - Coatings tribology: Properties, mechanisms, techniques and applications in surface engineering (2nd edition) (Elsevier Science), 2009, ISBN: 9780444527509

[36] Koren. Y.: Flank wear model of cutting tools using control theory, *J. Eng. Ind.,* 1978, **100**(1), 103–109, DOI: 10.1115/1.3439336

[37] Roumesy, B.: Détermination des lois d'usure des outils de coupe, *Machine-Outil*, 1975, **40**, 269–279

[38] ISO 3685:1993: Tool-life testing with single-point turning tools

[39] Denkena, B.; Tönshoff, H.K.: Spanen – Grundlagen (Springer Berlin, Heidelberg), 2011, DOI: 10.1007/978-3-642-19772-7

[40] Astakhov, V.P.: Tribology of cutting tools, in: Tribology in manufacturing technology, Davim, J.P. (Ed.) (Springer Berlin, Heidelberg), 2012, pp. 1–66, DOI: 10.1007/978-3-642-31683-8_1

[41] Ceratizit, S.A.: Full process control with toolscope https://cdn.plansee-group.com/is/content/plansee/md_sb_toolscope-flyer-a4_sen_asc_pim (accessed: 08/19/2024)

[42] Today's Medical Developments: Komet Brinkhaus toolscope monitoring system https://www.todaysmedicaldevelopments.com/product/komet-brinkhaus-toolscope-manufacturing-monitoring-31417 (accessed: 08/19/2024)