HUNGARIAN JOURNAL
OF INDUSTRIAL CHEMISTRY
VESZPRÉM
Vol. 29. pp. 113 – 117 (2001)

OPTIMIZATION OF PIPELINE NETWORK FOR OIL TRANSPORT

A. JOKIĆ and Z. ZAVARGÓ

(Faculty of Technolgy, University of Novi Sad, Bul Cara Lazara 1, 21000 Novi Sad, YU)

Received: June 18, 2001

In this work the pipeline network for oil transport has been optimized. The network layout has already been given, the flow rates are specified and the inlet and outlet pressures are defined. The capital cost of such a network has been minimized. From many feasible combinations of section diameter distributions the aim was to find the optimal one. The problem has been solved by applying nonlinear programming.

Keywords: pipeline network, oil transport, capital cost, optimization, nonlinear programming

Introduction

There are many works dealing with the solution of fluid network problems [1-3, 6, 8, 9]. There are also numerous algorithms for pipeline network optimization by minimizing some cost functions [6, 4]. The network which has to be optimized is the network for oil transport (*Fig.1*). The network consists of nodes, sections and paths. A pipe section is a pipeline with a constant diameter and no branching. A node is defined as the branching point or the point of inlet or outlet from the section. Finally, a path is defined as the sequence of pipe sections between an inlet (source) and outlet (consumer) from the network. Thus, each section has two nodes, one at each end. Each path has at least one section. The configuration of the network is given so are the flows through each section as well as the inlet and outlets pressures. While the inlet and outlet pressures are fixed, the other node pressures are unknown and subject to change via *D*. This problem has many feasible solutions. The goal was to find that one which minimizes the capital cost.

Pipe section flow

The oil that flows through the network can be considered as isothermal and incompressible. As the network is in the ground the temperature variances are negligible. Having in mind that the section diameter is constant, the section velocity is also constant. The Bernoulli equation for each pipe section in that case is

$$h_1 + \frac{P_1}{\rho g} - h_L = h_2 + \frac{P_2}{\rho g} \tag{1}$$

The total head loss is

$$h_L = \lambda \frac{v^2}{2g} \frac{L + L_e}{D} \tag{2}$$

where λ is the Darcy friction factor, while L and L_e are pipe length and equivalent length respectively. For a long distance pipeline the effect of L_e can be neglected. For hydraulically smooth pipes the friction factor depends only on Reynolds number. In that case Blasius correlation [5] can be used

$$\lambda = 0.3164 Re^{-0.25} \tag{3}$$

Combining *Eq. (1)* and *(2)* with continuity equation

$$v = \frac{4Q}{D^2 \pi} \tag{4}$$

it follows for the case when Blasius *Eq. (3)* can be used

$$K D^{-4.75} = -\Delta P - \Delta h \rho g \tag{5}$$

where

$$K = 0.242 Q^{1.75} \rho^{0.75} \mu^{0.25} L \tag{6}$$

and $\Delta P = P_2 - P_1$; $\Delta h = h_2 - h_1$.

Table 1 Pipeline network nodes data

Node	h[m]	P[kPa].10^{-5}
1	-	147.1
2	-	unknown
3	-	unknown
4	-	unknown
5	-	4.9
6	-	unknown
7	-	4.9
8	-	4.9
9	-	unknown
10	-	unknown
11	942	unknown
12	-	4.9
13	-	4.9

Table 2 Pipeline network sections data

Section	Input, output node	L[m]	Q[m^{3}s^{-1}]
S1	1,2	12,000	1.2688
S2	2,3	18,000	1.2688
S3	3,4	150,000	1.2688
S4	4,5	5,000	0.2985
S5	4,6	110,000	0.4478
S6	6,7	65,000	0.0746
S7	6,8	5,000	0.3732
S8	4,9	141,000	0.5224
S9	9,10	5,000	0.1866
S10	9,11	161,000	0.3358
S11	11,12	5,000	0.1119
S12	11,13	85,000	0.2239

Most of the pipes in the engineering practice have rough boundaries. In that case the equation of Altshul [5] can be used

$$\lambda = 0.11\left(\frac{\varepsilon}{D}+\frac{68}{\text{Re}}\right)^{0.25} \qquad (7)$$

where ε is the pipe surface roughnes. In this case the D cannot be explicitly expressed as it can be in Blasius *Eq. (3)*. The *Eqs. (5)* and *(6)* have now the following form

$$K^{*}D^{-5} = -\Delta P - \Delta h\rho g \qquad (8)$$

$$K^{*} = \frac{8Q^{2}\lambda\rho L}{\pi^{2}} \qquad (9)$$

Note that K isn't a true constant because it depends on D, since λ depends on D.

Pipe cost function

The weight per unit length of the pipe is assumed to be an exponential function of the diameter

$$W = \alpha D^{\beta} \qquad (10)$$

The total weight of the pipe section will be WL *[kg]* and the price of such a pipe can be easily calculated knowing the price of the pipe per unit weight. It is a well known fact that the ratio δ/D (pipe wall thickness/pipe diameter) depends upon pressure. Eq. (10) is usually correlated with a given pressure (P_{corr}). In the case of a pipe network it is usually a maximum expected pressure in the network. The higher pressure demands larger σ and consequently a larger weight of the pipe. When the pressure in the pipe (P) differs from pressure (P_{corr}) for which *Eq. (10)* was correlated, the following relation is proposed [7].

$$W = \alpha D^{\beta}\frac{P}{P_{corr}} \qquad (11)$$

Pipe network optimization

For a pipe network that consists of n pipe section there is an n *Eqs.(8)*

$$K_i D_i^{-4.75} = -\Delta P - \Delta h\rho g \qquad i = 1,n \qquad (12)$$

with $2n$ unknown variables (D_i and ΔP_i). To reduce the number of the equations as well as the number of the unknown variables the alternative system of equations can be formulated. For each path we have

$$\sum_{i=1}^{k} K_i D_i^{-4.75} = -\sum_{i=1}^{k}(\Delta P_i - \Delta h_i \rho g) = \Delta P_j - \Delta h_j \rho g \qquad j = 1,m \qquad (13)$$

where k corresponds to the number of the section that belongs to path j, while m represents the number of paths. In such a way the unknown pressure drops ΔP_i are excluded Recall that the paths pressure drops ΔP_j, from the source to the sink, are fixed.
The capital cost function which has to be minimized is

$$WN = \sum_{i=1}^{n} W_i L_i = \sum_{i=1}^{n} \alpha L_i D_i^{\beta} = \sum_{i=1}^{n} c_i D_i^{\beta} \qquad (14)$$

By introducing new variables into the *Eqs.(13)* and *(14)*, in the case when Blasisus relation can be used,

$$x_i = D_i^{-4.75} \qquad i = 1,n \qquad (15)$$

we have the following nonlinear cost function

$$(\min) WN = \sum_{i=1}^{n} c_i x_i^{-0.2105\beta} \qquad (16)$$

subject to linear constrains

$$\sum_{i=1}^{k} K_i x_i = \Delta P_j - \Delta h_j \rho g \qquad j = 1,m \qquad (17)$$

where $c_i = \alpha L_i$.
For rough pipes the following variables are introduced

$$x_i^{*} = D_i^{-5} \qquad i = 1,n \qquad (18)$$

Eqs. (14) and *(13)* are now

Table 3 Pipeline network paths data

Path	Sections	Input, output node	$-\Delta P$[kPa]10^{-6}
P1	S1,S2,S3,S8,S10,S12	1,13	14.22
P2	S1,S2,S3,S8,S10,S11	1,12	14.22
P3	S1,S2,S3,S8,S9	1,10	14.22
P4	S1,S2,S3,S4	1,5	14.22
P5	S1,S2,S3,S5,S6	1,7	14.22
P6	S1,S2,S3,S5,S7	1,8	14.22

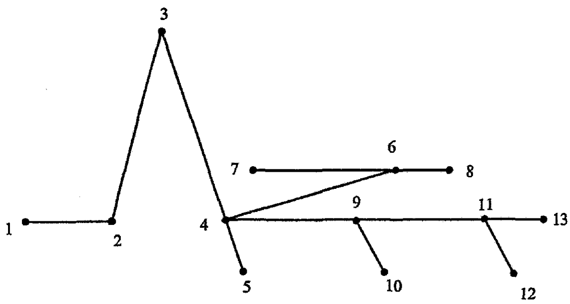

Fig.1 Pipeline network layout

$$(\min) WN = \sum_{i=1}^{n} c_i (x_i^*)^{-0.2\beta} \qquad (19)$$

subject to

$$\sum_{i=1}^{k} K_i^* x_i^* = -\Delta P_j - \Delta h_j \rho g \qquad (20)$$

Experimental

The layout of the pipeline network for oil transport is shown in *Fig.1*. The data necessary for the calculations are given in *Tables 1, 2* and *3* [7].

Hydraulically smooth pipes

For such systems the linear constrains are *Eqs. (17)*

$$\begin{aligned}
&K_1x_1 + K_2x_2 + K_3x_3 + K_8x_8 + K_{10}x_{10} + K_{12}x_{12} = 14{,}220{,}000\\
&K_1x_1 + K_2x_2 + K_3x_3 + K_8x_8 + K_{10}x_{10} + K_{11}x_{11} = 14{,}220{,}000\\
&K_1x_1 + K_2x_2 + K_3x_3 + K_8x_8 + K_9x_9 = 14{,}220{,}000\\
&K_1x_1 + K_2x_2 + K_3x_3 + K_4x_4 = 14{,}220{,}000\\
&K_1x_1 + K_2x_2 + K_3x_3 + K_5x_5 + K_6x_6 = 14{,}220{,}000\\
&K_1x_1 + K_2x_2 + K_3x_3 + K_5x_5 + K_7x_7 = 14{,}220{,}000
\end{aligned} \qquad (21)$$

and the objective function is

$$(\min) WN = \sum_{i=1}^{n} c_i x_i^{-0.2105\beta}$$

Thus we have a nonlinear objective function with linear constrains. *Eq. (11)* has been correlated for $P_{corr} = 159.10^5$ [Pa] and the obtained values for coefficients are: $\alpha = 1412.15$ and $\beta = 2$. The initial guess values for D as well as K are given in *Table 4* while the optimization results are presented in *Table 5*.

Table 4 Initial values for smooth pipes

Section	D[m]	K 10^{-6}
S1	1.0	0.463
S2	1.0	0.694
S3	1.0	5.782
S4	0.3	0.015
S5	0.6	0.685
S6	0.5	0.018
S7	0.4	0.023
S8	0.7	1.150
S9	0.3	0.007
S10	0.7	0.606
S11	0.3	0.003
S12	0.6	0.157

Table 5 Optimization results for smooth pipes

Section	D[m]
S1	1.025
S2	1.025
S3	1.025
S4	0.268
S5	0.624
S6	0.393
S7	0.414
S8	0.790
S9	0.255
S10	0.706
S11	0.272
S12	0.638

Pipes with rough boundaries

In this case the linear constrains are given by *Eqs. (20)*

$$\begin{aligned}
&K_1^*x_1^* + K_2^*x_2^* + K_3^*x_3^* + K_8^*x_8^* + K_{10}^*x_{10}^* + K_{12}^*x_{12}^* = 14{,}220{,}000\\
&K_1^*x_1^* + K_2^*x_2^* + K_3^*x_3^* + K_8^*x_8^* + K_{10}^*x_{10}^* + K_{11}^*x_{11}^* = 14.220.000\\
&K_1^*x_1^* + K_2^*x_2^* + K_3^*x_3^* + K_8^*x_8^* + K_9^*x_9^* = 14{,}220{,}000\\
&K_1^*x_1^* + K_2^*x_2^* + K_3^*x_3^* + K_4^*x_4^* = 14{,}220{,}000\\
&K_1^*x_1^* + K_2^*x_2^* + K_3^*x_3^* + K_5^*x_5^* + K_6^*x_6^* = 14{,}220{,}000\\
&K_1^*x_1^* + K_2^*x_2^* + K_3^*x_3^* + K_5^*x_5^* + K_7^*x_7^* = 14{,}220{,}000
\end{aligned} \qquad (22)$$

with the following objective function

$$(\min) WN = \sum_{i=1}^{n} c_i (x_i^*)^{-0.2\beta}$$

The initial guess values for D and K^* are presented in *Table 6* while the optimization results are given in *Table 7*. The value for $\varepsilon = 0.2$ is taken from literaure [11]. Since K^* values depend on D (see *Eqs. 9* and *7*) the optimization results cannot be achieved in one step. This means that on the base of the first optimum D values we have to recalculate K^* values and repeat the optimization procedure. As it can be seen (*Table 5* and

Table 6 Initial values for rough pipes

Section	D[m]	$K^{*}10^{-6}$
S1	1	0.464
S2	1	0.696
S3	1	5.803
S4	0.268	0.011
S5	0.62	0.613
S6	0.47	0.015
S7	0.5	0.019
S8	0.79	1.085
S9	0.26	0.005
S10	0.72	0.560
S11	0.3	0.021
S12	0.7	0.144

Table 7) the optimization results are practically the same as in the case of smooth pipes.

Conclusion

In this work the capital cost of pipeline network for oil transport has been minimized. The configuration of the network was fixed, so were the flow rates through each section as well as the pressure drops in each path. The fluid that flows through the network assumed to be isothermal and incompressible. The objective function which has to be minimized was the equation for the weight of the pipeline network as the function of section diameter D, while the constrains were equations for the pressure drop for each path. Diameter D of each section has been adjusted in such a way that it could give a minimum weight under the given constrains. The originally nonlinear constrains, with respect to D, were linearized introducing new variables. The objective function remains nonlinear.. The linearly constrained nonlinear objective function has been solved by nonlinear programming. Two relations for Darcy friction factor λ have been used. Blasius formula for hydraulically smooth pipes, *Eq. (3)*, and Altshul correlation for rough pipes, *Eq. (7)*. In the latter, due to the fact that λ is function of D and K is function of λ, K has to be recalculated in each step of the procedure.

SYMBOLS

c	coefficient in *Eq. (16)*
D	pipe diameter, m
g	acceleration due to gravity, kgm^{-2}
h	height, m
Δh	height difference, m
h_L	head loss, m
i	index for pipe network sections
j	index for pipe network paths
k	number of sections in a given path
L	node height, m
L_e	equivalent height, m
m	number of pipe network paths
n	number of pipe network sections
P	pressure, kPa
ΔP	pressure drop, kPa
P_{corr}	pressure for which *Eq. (11)* has been correlated, kPa
Q	Flow rate, m^3s^{-1}
Re	Reynolds dimensionless number
W	weight per unit length, kgm^{-1}
WS	one section weight, kg
WN	whole network weight, kg
x	variable defined by *Eq. (15)*
x^*	variable defined by *Eq. (18)*

Table 7 Optimization results for rough pipes

Section	D[m]	$K^{*}10^{-6}$
S1	1.025	0.467
S2	1.025	0.701
S3	1.025	5.838
S4	0.268	0.011
S5	0.624	0.614
S6	0.393	0.014
S7	0.414	0.018
S8	0.79	1.085
S9	0.255	0.005
S10	0.706	0.557
S11	0.272	0.020
S12	0.638	0.141

Greek symbols

α	coefficient in *Eq. (10)*, kg
β	coefficient in *Eq. (10)*
δ	pipe wall thickness, m
λ	Darcy friction factor

Abbreviations

P1, P2,… path 1, 2,…
S1, S2, … section 1, 2, ..

REFERENCES

1. BENDING M. J. and HUTCHISON H. P.: Chem.Engng.Sci., *1973*, 28, 1851-1864
2. BULLARD L. G. and BIEGLER L. T.: Comput.Chem.Engng., *1991*, 15, 239-254
3. GAY B. and MIDDLETON P.: Chem.Engn.Sci., *1971*, 26, 109-123
4. DRAGUTINOVIĆ G.: M. S. thesis, Faculty of Technical Science, Novi Sad, 1991
5. GULIANI B. B. and AGAURWAL C. P.: PTQ, Autumn *2000*, 129-131
6. MURTAGH B. A.: Chem.Engng.Sci., *1972*, 27, 1121-1141
7. PEJAK M.: B. S. thesis, Faculty of Technology, Novi Sad., 1979

8. PREECE P. E. and HWEI CHEN TI: The Chem,Engng.J., *1989*, 40, 1-6
9. SHACMAN M. and MAH R. S. H.: Comput.Chem.Engng., *1978*, 2, 64-66
10. WELTY J. R., WICKS E. E. and WILSON R. E.: Fundamentals of momentum, heat and mass transfer, John Wiley & Sons, New York, 210-211, *1976*
11. ŠAŠIĆ M.: B. S. thesis, Faculty of Mechanical Science, Beograd, 1990