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MATHEMATICAL MODELS FOR A CLOSED-CIRCUIT GRINDING

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New continuous and discrete mathematical models are elaborated for describing some types of closed-circuit grinding mill-classifier systems. Based on the discrete model computer simulation is also developed for investigation of grinding processes.

The starting point in developing the model is the continuous grinding equation for the open-circuit grinding which is a partial integro-differential equation describing axial mixing and breakage of particles in the grinding mill. The convective flow in the mill is modeled as particle size-dependent process. The boundary conditions at the inlet and outlet of the mill, the initial conditions, and the equations describing the operation of the classifier and its mass balance are the additional equations of the model.

The properties and capabilities of the new computer model are demonstrated and analyzed by simulation developed in Matlab environment. The effects of all parameters of the grinding process, among others of parameters of the classifier, are studied via simulation. The main statistical characteristics of the steady states, the hold-up of the grinding device and the operation of the classifier are also studied.

The models presented appear to be very flexible and useful tools for analysis and design of closed-circuit grinding processes, and are suitable for the deeper understanding of the aimed processes.

Keywords: Closed-circuit grinding, computer simulation, grinding-classifying system, mathematical modeling, mill-classifier system.

Introduction

Grinding is a widely used operation in the industrial processes. Due to the fact that there are numerous types of grindings many kinds of grinding devices have been developed based on various operation principles. At the same time, grinding is an energy consuming operation therefore the cost-efficiency of the operation can be improved by the decreasing the grinding energy. One of the main urgings of the mathematical description and modeling of grinding is exactly the improvement of the cost-efficiency.

Numerous excellent papers have been devoted to the description of the batch grindings [1, 2, 3]. Mihálykó, Blickle and Lakatos [4] developed deterministic continuous and discrete models for the open circuit grinding describing processes in long ball mills. Further development and generalization of these models provided mathematical description also for closed-circuit grinding systems with classifying devices [5, 6]. In the literature, grinding with internal classification of the product has also been modeled [7, 8].

The aim of this paper is to present continuous and discrete mathematical models for describing some further types of closed-circuit grinding mill-classifier systems. The model is formed by a continuous grinding equation for the open-circuit grinding process, being a partial integro-differential equation describing axial mixing and breakage of particles in the mill. The convective flow of ground material in the mill is modeled as size-dependent process. The boundary conditions at the inlet and outlet of the mill, the initial conditions, and the equations describing the operation of the classifier and its mass balance are the additional equations of the model.

Using the discrete model computer simulation is carried out investigating the effects of parameters and operational conditions of the grinding system.

Continuous and discrete mathematical models

The closed-circuit grinding models of the authors which have been previously published describe such a grinding-classifying system which returns the large particles from the classifier to the inlet of the mill [5, 6, 9].

This paper presents mathematical models referring to another group of the closed-circuit grindings. The considered process is as follows: the fresh particles to be ground are entered into the mill through the inlet of the mill; the particles are grinding while they are flowing along the mill; the particles are classified; only the fine particles are allowed to leave the mill through the outlet; due to the classification the large particles are retained in the mill.

Let symbol x stands for the particle size, let symbol x_{max} denote the largest particle size. The convective flow of the particles along the device is characterized via the convective flow velocity and denoted by $u(x)$, while symbol D stands for the axial dispersion of the particles. The operation of the classifier is characterized by the classification function $\psi(x)$. Let the mass density function $m(x, y, t)$ characterize the size distribution of the particles where $m(x, y, t)\mathrm{d}x$ expresses the mass of the particles at the axial coordinate y of the mill at time t within the particle size interval $(x, x+\mathrm{d}x)$ in a unit mass of the particles. At the model-construction let us suppose that 1) the transport of the particles in the mill is described by the axial dispersion model; 2) the grinding kinetics is described by the first order law of breakage.

The continuous mathematical model is given by the following equations (1) – (6). The operation of the grinding mill is described via equation (1) which is a partial integral-differential equation:

$$\frac{\partial m(x,y,t)}{\partial t} = D\frac{\partial^2 m(x,y,t)}{\partial y^2} - u(x)\frac{\partial m(x,y,t)}{\partial y}$$

$$- S(x)m(x,y,t) + \int_x^{x_{\max}} S(L)b(x,L)m(L,y,t)\mathrm{d}L. \quad (1)$$

Equations (2) and (3) – (4) express the initial and the boundary conditions, respectively:

$$m(x,y,0) = m_0(x,y), \quad (2)$$

$$f_f(x,t) = u(x)\cdot m(x,y,t) - D\cdot\frac{\partial m(x,y,t)}{\partial y}, \quad \text{if } y = 0, \quad (3)$$

$$\frac{\partial m(x,y,t)}{\partial y} = 0, \ \text{if } y = Y. \quad (4)$$

In equation (1) function $S(L)$ represents the rate of breakage of particles of size L , usually termed selection function, while function $b(x, L)$ is the breakage density function. In equation (3) function $f_f(x, t)$ denotes the mass flow rate of the feed to the mill. Equation (3) expresses the assumption that there is no back-mixing from the mill into the transfer pipe. Symbol Y stands for the length of the mill, therefore $y = 0$ expresses the inlet of the mill in equation (3) and $y = Y$ means the outlet of the mill in equation (4).

Equations (5) and (6) describe the operation of the classifier. The mass flow of the large particles is given by equation (5):

$$f_r(x, t) = \psi(x)\cdot f(x, t) = \psi(x)\cdot u(x)\cdot m(x, Y, t). \quad (5)$$

In equation (5) $f(x, t)$ is the mass flow density function at the outlet of the mill. We suppose that the large particles – which are not allowed to leave the mill due to the operation of the classifier – remain in the grinding device. This assumption is expressed by equation (6):

$$f(x, t) = f_l(x, t) + \psi(x)\cdot f(x, t), \quad (6)$$

where function $f_l(x, t)$ is the mass flow density function of that particles which flowed in the grinding device to the outlet of the mill.

Using the continuous model – equations (1) – (6) – a discrete mathematical model has been derived similar way involved in the paper [6].

Let Δt denote the length of the time step, let t_n stand for $t_n = n\Delta t$. Let us subdivide the mill into J equal sections and denote h_y the length of a section of the mill, thus $h_y = Y/J$. Let us sort the particles into size fractions. Let x_{min} denote the smallest particle size. If I denotes the total number of size fractions of particles and we introduce the notation $h_x = (x_{max} - x_{min})/I$, then $x_i = x_{min} + ih_x$ stands for the i^{th} particle size fraction. Let symbol $\bar{x}_i$ denote the mean size of the i^{th} size fraction, i.e. $\bar{x}_i = (x_{i-1} + x_i)/2$ and let $\bar{t}(\bar{x}_i)$ stand for the mean residence time of particles belonging to the i^{th} size interval, i.e. $\bar{t}(\bar{x}_i) = Y/u(\bar{x}_i)$. Let $\mu(x_i, y_j, t_n)$ denote the mass of the particles belonging to the i^{th} size fraction and the j^{th} section at the moment of time t_n.

Let us introduce symbols $V_F(\bar{x}_i)$ and $V_B(\bar{x}_i)$ (i=1, 2, …, I) via the following formulae:

$$V_F(\bar{x}_i) = \frac{\tau(\bar{x}_i)\left(1+\dfrac{Pe(\bar{x}_i)}{J}\right)}{\dfrac{Pe(\bar{x}_i)}{J^2}},$$

$$V_B(\bar{x}_i) = \frac{\tau(\bar{x}_i)}{\dfrac{Pe(\bar{x}_i)}{J^2}},$$

where $Pe(\bar{x}_i) = u(\bar{x}_i)Y/D(\bar{x}_i)$ is the Peclet number for the i^{th} size fraction of particles. In the above formulae $u(\bar{x}_i)$ and $D(\bar{x}_i)$ denote the convective flow velocity and the axial mixing coefficient of that size fraction, respectively, while $\tau(\bar{x}_i) = \Delta t/\bar{t}(\bar{x}_i)$.

The breakage distribution function – let it denote symbol $B(X, x)$ – describes of the rubbles when the particle of size X breaks. The connection between the breakage distribution function and the density function is as follows:

$$B(X,x) = \int_{x_{\min}}^{x} b(l,X)dl .$$

After introducing the notation

$$p_{k,i} = \Delta t S(\bar{x}_k)[B(\bar{x}_k, x_i) - B(\bar{x}_k, x_{i-1})]$$

then the equations of the discrete model can be described by equations (7) – (9). The development of the discrete model can be manipulated similarly to those published in the paper [6].

The left-hand sides of equations (7), (8), (9) contain the mass of the particles belonging to the i^{th} size fraction at the moment of time t_{n+1}. The first group of equations describes the size composition of particles in the first section; the second group reflects to the inner sections of the mill; finally, equation (9) expresses the size composition in the last section of the mill.

$$\mu(x_i, y_1, t_{n+1}) = \\ = (1 - V_F(\bar{x}_i) - \tau(\bar{x}_i)\bar{t}(\bar{x}_i)S(\bar{x}_i))\mu(x_i, y_1, t_n) + \\ + V_B(\bar{x}_i)\mu(x_i, y_2, t_n) + \\ + \sum_{k=i}^{I} p_{k,i}\mu(x_k, y_1, t_n) + a_i(t_n) \quad i=1, 2, \ldots, I. \quad (7)$$

The first term on the right-hand side of equation (7) expresses the mass of particles belonging to the i^{th} size interval at the moment of time t_n which neither moved forward nor broke; the second term represents that particles of the same size interval that moved backwards from the second section; the third term gives the mass of particles that remained in the first section and broke from some greater or equal size than x_i to this very size interval; the last term denotes the mass of the feed particles belonging to the i^{th} size interval.

$$\mu(x_i, y_j, t_{n+1}) = \\ = (1 - V_F(\bar{x}_i) - V_B(\bar{x}_i) - \tau(\bar{x}_i)\bar{t}(\bar{x}_i)S(\bar{x}_i)) \\ \times\mu(x_i, y_j, t_n) + V_F(\bar{x}_i)\mu(x_i, y_{j-1}, t_n) + \\ + V_B(\bar{x}_i)\mu(x_i, y_{j+1}, t_n) + \\ + \sum_{k=i}^{I} p_{k,i}\mu(x_k, y_j, t_n) \quad j=2, \ldots, J-1, \; i=1, 2, \ldots, I. \quad (8)$$

The first term on the right-hand side of equation (8) gives the mass of particles of the i^{th} size fraction that did not changed at all during a unit of time; the second term represents the fraction of particles belonging to the i^{th} size interval that moved forward from the previous section; the third term expresses that part of particles which moved backward from the next section; while the last term means the mass of those particles that remained in the section and broke from some size greater or equal than x_i to the interval in question.

Finally, equation (9) describes the size composition of the particles as

$$\mu(x_i, y_J, t_{n+1}) = \\ = (1 - V_B(\bar{x}_i) - \tau(\bar{x}_i)\bar{t}(\bar{x}_i)S(\bar{x}_i))\mu(x_i, y_J, t_n) + \\ + V_F(\bar{x}_i)\mu(x_i, y_{J-1}, t_n) + \\ + \sum_{k=i}^{I} p_{k,i}\mu(x_k, y_J, t_n) + \\ + \psi(\bar{x}_i)(V_F(\bar{x}_i) - V_B(\bar{x}_i))\mu(x_i, y_J, t_n) \quad i=1, 2, \ldots, I. \quad (9)$$

where the first term on the right-hand side describes the mass of those particles which did not changed at all; the second term represents the particles which moved forward from the last but one section; the third term represents the mass of particles that remained in the last section and broke to the interval in question; the last expression, i.e. $\psi(\bar{x}_i)(V_F(\bar{x}_i) - V_B(\bar{x}_i))\mu(x_i, y_J, t_n)$ gives the mass of that particles of that size were retained in the mill due to the classification.

The set of recursive equations (7), (8), and (9) provide, in principle, the discrete mathematical model of the closed-circuit grinding system. In the model the constitutive relations $D(x)$, $u(x)$, and $\psi(x)$ are of arbitrary form, and can be formulated arranging the simulation in accordance with the equipment and operational conditions.

Numerical experiments and results

Based on the discrete mathematical model computer simulation has been elaborated in Matlab environment for examination the capabilities and properties of the model. The particle size distribution of the material to be ground was chosen from the literature [10] as well as the parameters in the model which are the kinetic and process parameters. The parameters of the selection function and breakage distribution function form the kinetic parameters, while the convective flow velocity, the axial dispersion, and the parameters of the classification function constitute the process parameters.

The forms of the selection function and breakage distribution function are as follows:

$$S(x) = K_s \cdot x^{\alpha},$$

where K_s and α are the parameters of the function;

$$B(X, x) = \Phi \cdot \left(\frac{x}{X}\right)^{\gamma} + (1 - \Phi) \cdot \left(\frac{x}{X}\right)^{\beta},$$

where β, γ, Φ are the parameters [11].

The size dependency of the convective flow velocity of particles was characterized via the formulae

$$u(x_i) = k_i \cdot u(x_1), \quad (i=1, 2, \ldots, I)$$

where

$$k_i = \frac{1}{1 + (x_i / x_{50})^{\lambda}} \quad 0 \le k_i \le 1,$$

where $u(x_1)$ stands for the velocity of flow of the finest particles. In the formula for k_i symbol x_{50} denotes that particle size for which the convective flow velocity is $0.5u(x_1)$, and λ is an appropriate constant [12].

The classifier was modeled using the Molerus curve approach so the operation of the classifier was characterized by the following efficiency curve [13]:

$$\psi(x) = 1 - \frac{1}{1 + \frac{x}{x_{cut}} \cdot e^{-c(1 - \frac{x}{x_{cut}})}}.$$

In the formula for $\psi(x)$ symbol x denotes the particle size. Function $\psi(x)$ has two parameters; c and x_{cut}.

Parameter c characterizes the classifier, while x_{cut} means the cut size. It is very likely that the classifier retains a particle in the mill if the size of a particle is greater than the cut size. Parameter c expresses the sharpness of the cut.

With the aid of the computer simulation the operation of the grinding-classifying system was investigated via numerical experiments. The effects of both the kinetic and process parameters were examined. The statistical characterization of the ground material at the steady state was also performed.

Let us see an example, i.e. an experiment which demonstrates the effect of the classification to the steady state characteristics of the material at the outlet from the mill. The aim of the numerical experiment is to show the influence of the cut size of the classifier to the mean particle size and dispersion of the particle size distribution of the ground material which leaves the mill at the outlet. The calculation results are involved in the *Table 1* and *Table 2*. In the numerical experiments the value of the parameter c was 5 and 10, respectively. The grade efficiency curves of the classifier is illustrated by *Fig. 1*, while *Fig. 2* shows the distribution of the feed material and the distribution of the ground material at the outlet from the mill at the steady state.

The values of the remaining parameters were: $Y = 5.4$ m, $\alpha = 1.2$, $\beta = 2.6$, $\gamma = 0.8$, $\Phi = 0.4$, $K_s = 0.028$ s^{-1}, $x_{min} = 0$ µm, $x_{max} = 1540$ µm, $u(x_1) = 0.090$ m/s, $D = 0.002$ m^2/s, $\lambda = 3.5$, $x_{50} = 650$ µm.

Table 1: The steady state characteristics of the material at the outlet from the mill depending on the cut size of the grade efficiency curve, where $c = 5$

Cut size (µm) (c = 5)	**The steady characteristics of the material at the outlet from the mill**	
	Mean particle size (µm)	**Dispersion of the particle size distribution (µm)**
200	136	103
300	171	138
400	191	162

Table 2: The steady state characteristics of the material at the outlet from the mill depending on the cut size of the grade efficiency curve, where $c = 10$

Cut size (µm) (c = 10)	**The steady characteristics of the material at the outlet from the mill**	
	Mean particle size (µm)	**Dispersion of the particle size distribution (µm)**
200	117	83
300	155	119
400	180	147

Table 1 and Table 2 illustrate well how the average particle size and dispersion at the outlet from the mill keep growing with increasing the cut size of the classifier. At the same time, the tables show the effect of the sharpness of the classification. The greater is the value of parameter c the sharper is the classification function, i.e. the better is the classification. In the case of a good classification the large particles have a very little chance to leave the mill. Therefore the greater value of the parameter c yields smaller mean particle size and dispersion of the particle size distribution. This fact is also demonstrated by the data involved in the Table 1 and Table 2.

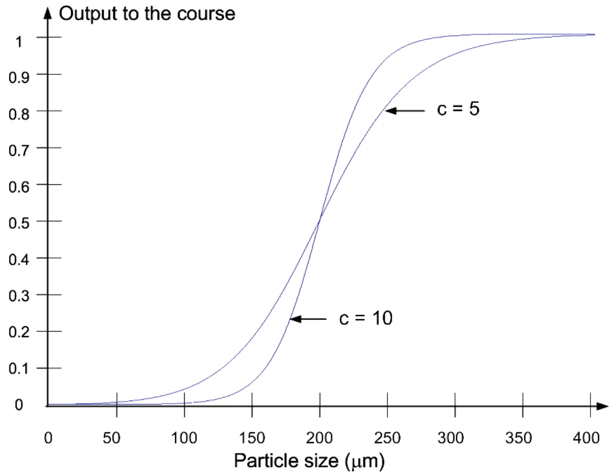

Figure 1: Grade efficiency curves of the classifier, $c = 5$ and $c = 10$

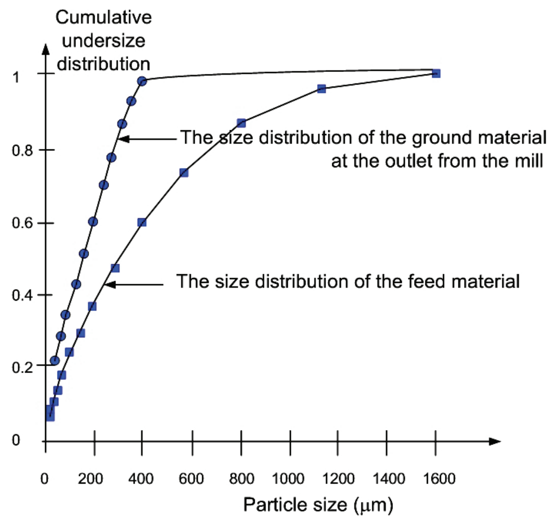

Figure 2: The size distribution of the feed material and the size distribution of the ground material at the outlet from the mill where $x_{cut} = 200$, and $c = 10$

The hold-up of the grinding device can also be investigated via numerical experiment. Let us see how the operation of the classifier and one of the kinetic parameters – K_s – influence the hold-up of the mill at the steady state. The effects of the cut size, sharpness of the cut, and parameter K_s were studied. It was supposed that the amount of the particles in the mill was a unit at the start of the grinding process. The results of the calculations are seen in the *Table 3* and *Table 4* where $K_s = 0.005$ s^{-1} and $K_s = 0.050$ s^{-1}, respectively. *Fig. 3* and *Fig. 4* also illustrate the hold-up of the mill at the steady state.

The values of the remaining parameters were: $Y = 5.4$ m, $\alpha = 1.2$, $\beta = 2.6$, $\gamma = 0.8$, $\Phi = 0.4$, $x_{min} = 0$ µm, $x_{max} = 2000$ µm, $u(x_1) = 0.090$ m/s, $D = 0.002$ m^2/s, $\lambda = 3.5$, $x_{50} = 650$ µm.

Comparing Table 3 and Table 4, the results indicate that the increase of the value of parameter K_s causes the decrease of the hold-up of mill.

Table 3: The hold-up of the grinding device at the steady state depending on the cut size and sharpness of the cut, where $K_s = 0.005\ s^{-1}$

Cut size (μm)	**The hold-up of the mill at the steady state**		
	Sharpness of the cut $c = 3$	**Sharpness of the cut** $c = 5$	**Sharpness of the cut** $c = 10$
220	2.8	3.8	3.8
240	2.4	3.7	3.8
260	**2.1**	3.1	3.8
280	1.8	2.7	3.8
300	1.7	2.4	3.8
320	1.5	2.1	3.4

Table 4: The hold-up of the grinding device at the steady state depending on the cut size and sharpness of the cut, where $K_s = 0.050\ s^{-1}$

Cut size (μm)	**The hold-up of the mill at the steady state**		
	Sharpness of the cut $c = 3$	**Sharpness of the cut** $c = 5$	**Sharpness of the cut** $c = 10$
220	**2.1**	2.9	3.7
240	1.8	2.5	3.7
260	1.6	2.2	3.2
280	1.5	2.0	2.8
300	1.4	1.8	2.5
320	1.3	1.6	2.3

Table 3 and Table 4 show well that the increase of the cut size induces the decrease of the hold-up for a fixed value of the sharpness of the cut generally; as well as the increase of the sharpness of the cut gives the increase of the hold-up for a fixed cut size mostly.

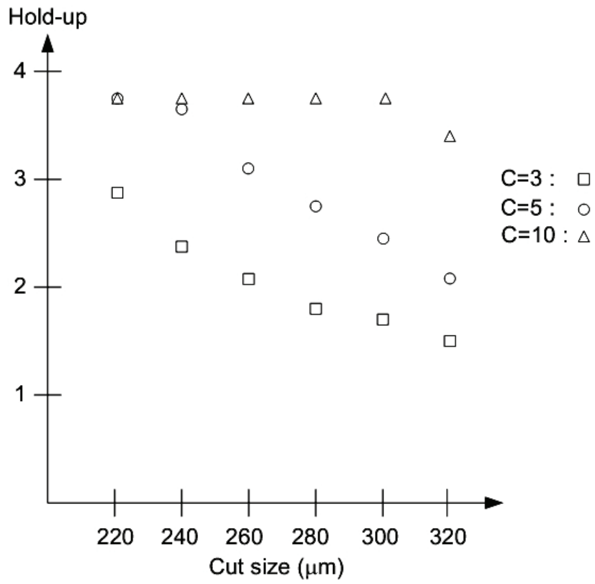

Figure 3: The hold-up of the grinding device at the steady state depending on the cut size and sharpness of the cut, where $K_s = 0.005\ s^{-1}$

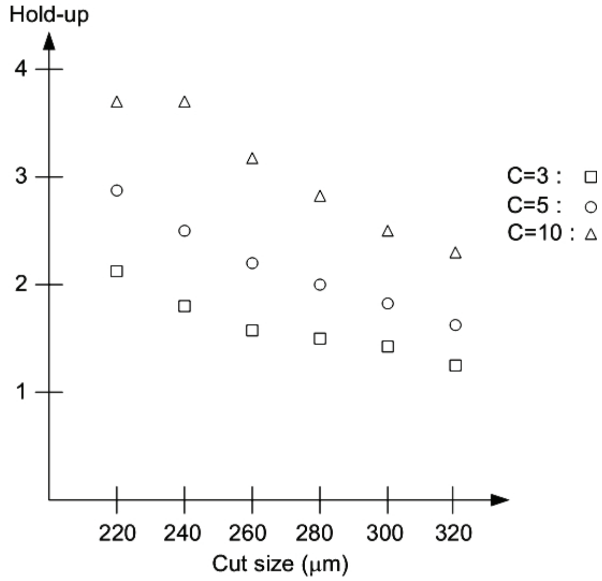

Figure 4: The hold-up of the grinding device at the steady state depending on the cut size and sharpness of the cut, where $K_s = 0.050 s^{-1}$

We can find some equal values of the hold-ups in Table 3 and Table 4 for just the same value of parameter c, for example, $c = 3$ and the hold-up is 2.1. *Table 5* contains the statistical characteristics at the outlet from the mill in both cases, i.e. $K_s = 0.005\ s^{-1}$ and $K_s = 0.050\ s^{-1}$. In order to produce equal hold-ups the higher value of the parameter K_s connects to smaller cut size. As a consequence the values of the mean particle size and the dispersion of the particle size distribution are also smaller as it is seen in Table 5.

Table 5: The values of the mean particle sizes and the dispersions of the particle size distribution where $c = 3$ and the hold-up is 2.1

Parameter K_s	**Cut size**	**Mean particle size**	**Dispersion**
0.005	260	222	180
0.050	220	193	153

Summary

Numerous papers have been devoted to the mathematical description of the batch grindings. At the same time the mathematical description and modeling of continuous grinding processes, in particular the closed-circuit grindings still offer a lot of work for the process engineers and experts who investigate this area. The newly elaborated continuous and discrete mathematical models proved to be suitable to the description and analysis of some types of closed-circuit mill-classifier systems.

In particular, the newly developed mathematical models presented in the paper involve the description of both the batch grinding and the open-circuit grinding systems. Notice that the discrete mathematical model of the closed-circuit grinding system, i.e. the set of recursive equations (7), (8), and (9), can also be written in matrix form. The matrix form of the model is suitable for some further, among others stability investigations.

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